



# *Computer Graphics And Multimedia*

*Contributed By:*  
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# *Computer Graphics And Multimedia*

Topic:  
*Introduction To Subject*

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# Computer Graphics and Multimedia Techniques

Introduction to subject :-

The computer graphics refers to anything involved in the creation & manipulation of images on computer.

Computer graphics is an art of drawing pictures on computer screens with the help of programming. It involves computation, creation & manipulation of data.

Application of Computer Graphics :-

CAD - 3D modeling of objects

Entertainment → Games, movies.

Virtual Reality.

Visualization for science & business.

Hospital's device for tracking patient's body.

Image processing.

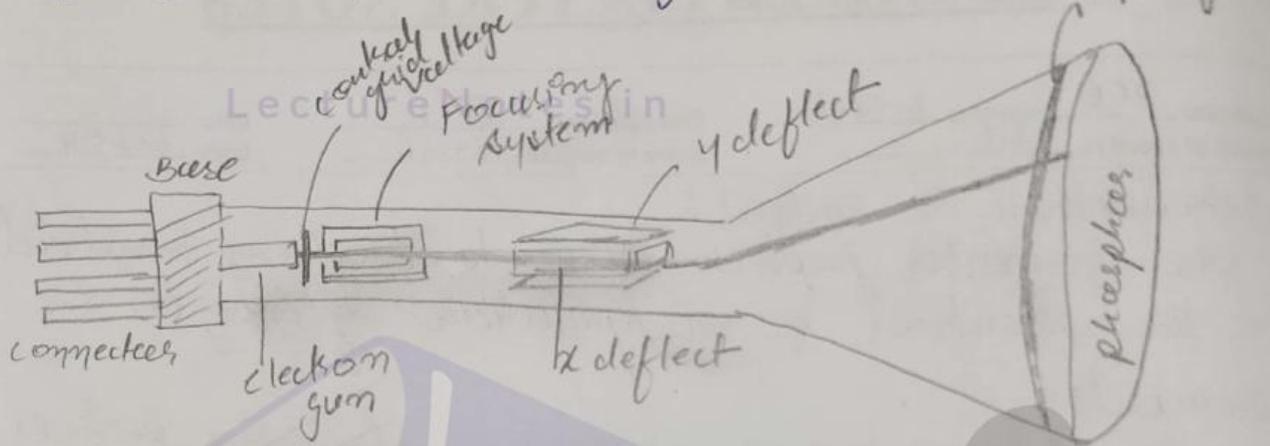


# *Computer Graphics And Multimedia*

Topic:  
*Video Display Device*

Contributed By:  
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video Display Devices:-  
 The primary output devices in graphics system is a video monitor. The operation of most of the devices is based on the standard cathode-ray tube (CRT).



operation of CRT:-

1. Electron gun emits a beam of electron.
2. Electron beam passes through focusing & deflecting system, that direct it towards the specified position on the phosphor coated screen.
3. when beam hits the screen, the phosphor emits a small spot of light at each position contacted by electron beam.
4. It redraws the picture by directing the electron beam back over the same screen points quickly.
5. Focusing system & x & y deflects are anodes, which are magnetized to push & pull the electrons in a specific direction.

6. There are three different color phosphors. (R G B) for each pixel & the color of pixel depends on the phosphor on which electron strikes.

7. Aperture grill is a metal sheet consisting of holes, which controls the diameter on the screen.

8. Different level of phosphors are available for CRT, but major difference b/w them is the persistence, how long they continue to emit light.

9. The max. no. of points that can be displayed without overlap on a CRT is referred as resolution.

10. Aspect Ratio :- This no. gives the ratio of vertical points to horizontal points necessary to produce equal length lines in both directions on screen.

Aspect ratio  $3/4$  means vertical line plotted with 3 points has same length as a horizontal line plotted with four points.



# *Computer Graphics And Multimedia*

Topic:  
*Types Of Display*

Contributed By:  
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There are two ways by which we can display a object on the screen:-

### 1 Raster Scan Display :-

1. The image which is to be displayed is scanned in a binary format of 0 & 1. in refresh buffer memory.
2. A video controller is used to scan each & every line of the Refresh Buffer memory.
3. Each line is scanned from left to right and the electron beam of video controller is turned on & off to create a pattern of illuminated spots.
4. The refresh buffer or frame buffer stores various information:-
  - a) coordinates of screen points
  - b) intensity values of screen points.
5. These information is retrieved from the buffer & printed on the screen one row at a time
6. Each point is referred as pixel or pel (picture element).
7. Printers are the example of raster scan displays.

8. In black & white system: one bit is needed to control the intensity of screen position.

for binary: 2 bits are needed. (0 & 1)

1 → beam intensity on  
0 → " " off.

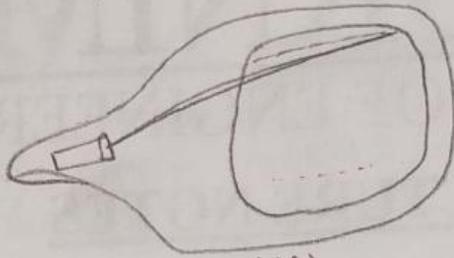
24 bits per pixel are included in high-quality systems.

24 bits per pixel system need screen resolution of  $1024 \times 1024$  are 3 MB of frame buffer.

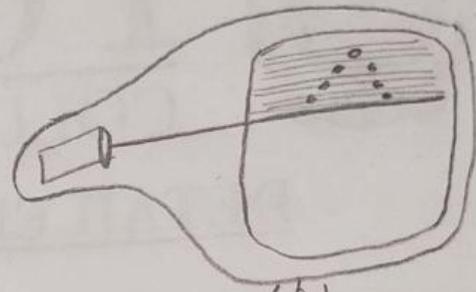
To refresh buffer, refresh rate is 60-80 frames per second.

After scanning all the horizontal lines, at the end of each frame, the vertical retrace returns to the top left corner of the screen to begin next frame.

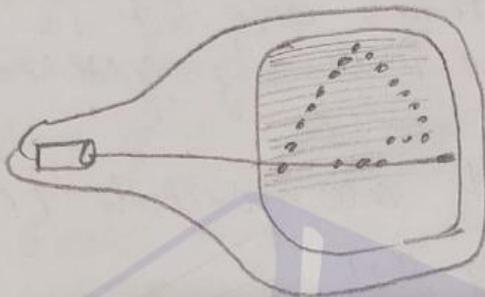
A raster scan system displays an object as a set of discrete points across each scan line.



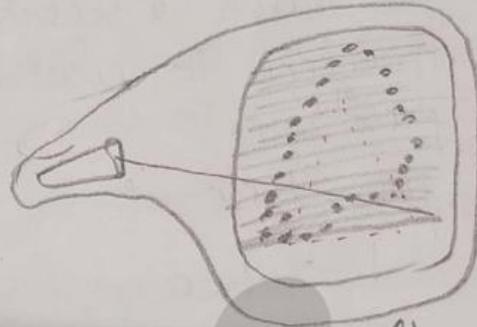
(a)



(b)



(c)



(d)

## 2o Random scan display

In this technique, the beam is directed only to the part of the screen where the picture is to be drawn rather than scanning from left to right.

The picture definition is stored as a set of line drawing commands in refresh display file (buffer). The video controller here, cycles through the set of commands in the display file, drawing each component line in turn. After all line drawing commands are processed, the system cycles back to the first line in the list. Designed to draw component lines of a picture 30 to 60 times each second.



# *Computer Graphics And Multimedia*

Topic:  
*Color Monitors*

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## Color Monitors

### Beam Penetration

### Shadow Masking

Color Monitors :-

There are 2 ways to produce color with CRT

① Beam penetration method :-

Two phosphors of Red & Green are coated inside of CRT screen.

The color displayed depends on how far electron beam penetrates into the layer.

Slow electron only penetrates to Red.

Excited electron penetrates to Green.

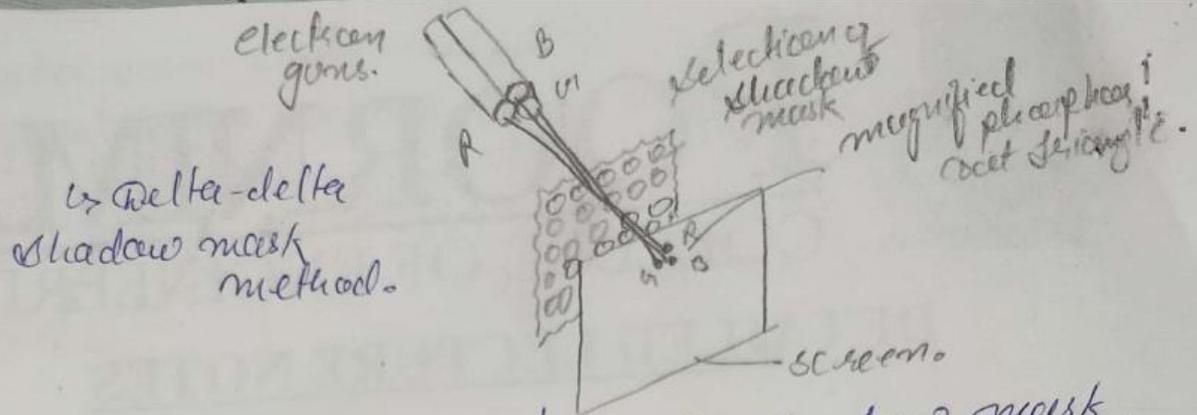
The speed of electrons is controlled by the beam acceleration voltage. With this only

4 colors are possible.

② Shadow mask :-

Produces wider range of colors.

↳ Three phosphors RGB emits R, G, B light & have three electron gun one for each light and a shadow mask grid just behind the phosphor coated screen.



When 3 beams pass through shadow mask, they activate a dot triangle, which appears as a small color spot on the screen.

Another method is, 3 electron guns are placed in line arrangement. This is commonly used in high resolution color CRTs.

Different intensity of colors can be obtained by turning on & off one or two of these electron guns.

### Storage tubes :-

Alternate method for maintaining a screen image is to store the picture info inside the CRT instead of refreshing the screen. A storage tube stores the picture info as a charge distribution just behind the phosphor coated screen.

Two electron guns are used :-

1 → primary gun - store picture pattern.

2 → flood gun - maintain picture display.

Disadv :- Do not display color, & can't be erased. The erasing & redrawing process can take several

## Difference between Raster and Random Scan Systems

Properties	Raster Scan System	Random Scan System
Resolution	less Resolution because picture definition is stored as a intensity value.	High Resolution picture definition is stored as a set of line of commands.
Electron Beam	directed from top to bottom & one row at a time on screen	directed to only that part of the screen where picture is required to be drawn.
Cost	less expensive	Cost lies.
Refresh Rate	60 to 80 fps	rate depends on no. of lines to be displayed 30-40 fps.
Picture Definition	Store picture definition on in Refresh buffer.	Stored as a set of line of commands in a Refresh display file.

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Properties	Raster Scan System	Random Scan system:
Line Drawing	Zig-zag line is produced because plotted values are discrete.	Smooth line is produced because direct line path is followed by electron beam.
Image Drawing	It uses pixels along scan lines for drawing an image.	It is designed for line drawing apps & uses various mathematical functions to draw.

### Difference b/w Beam Penetration & Shadow Masking.

Properties	Beam Penetration	Shadow Mask
where used	used with random scan system to display colors.	used with raster scan system to display colors.
Colors.	It can display only 4 colors, Red, green, orange & yellow.	It can display millions of colors.

## Difference between Beam Penetration and Shadow Masking

Properties	Beam Penetration	Shadow Mask
Color Dependency	Color depends on the level of electron penetration into the phosphorus layer.	Color depends on the type of ray.
Cost	less expensive	costlier.
Picture Quality	Not good	Good quality picture, realistic.
Resolution	High	low
Criteria	Color display depends on how far electron strikes the outer red layer & then green layer.	There is no such criteria for producing colors. It used in computer, TV.



# *Computer Graphics And Multimedia*

Topic:  
*Interlacing Techniques*

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## Interlacing Technique

**Interlacing** :- It is a description how the picture is created. In interlaced monitors, the electron beam takes two passes to form a complete image, it skips every other row on the first pass & then goes back & fills the missing rows. Interlaced monitors are easier to build & therefore cheaper. The problem is that all things being equal as non-interlaced monitors, it takes twice as long to create the complete screen image. That's long enough to spoil the illusion that you are looking at steady picture & the image on the screen flickers annoyingly. Interlaced monitors are needed because earlier, normal CRT display devices scan hundred horizontal lines in a frame, and the refresh rate is slower so, it causes headache & eye fatigue & also producing distracting screen flicker.

**Flickering** :- The appearance of flickering, flashing or unsteadiness in an image may occur

On a display screen. A flicker issue can occur when the refresh rate is too low, or other video related issues.

Refresh Rate :- It is the no. of times per second pixels are recharged so the image doesn't flicker.

A refresh rate of 60Hz means image is redrawn 60 times a second. The higher the refresh rate, the lower is the flickering.

A refresh rate below 75Hz can produce an often imperceptible flicker that can cause eye strain after long viewing.

Calculation of refresh rate :-

$$f_v = f_H / \# \text{ of horizontal lines} \times 0.95$$

$f_v$  - vertical scanning freq.

$f_H$  - horizontal scanning freq.

Ex:- a monitor with horizontal frequency of 96 kHz at a resolution 1280 x 1024 would have following refresh rate.

$$f_v = \frac{96 \times 1000}{1024 \times 0.95}$$

$$f_v = 89.06 \text{ Hz} = \underline{\underline{89 \text{ Hz}}}$$

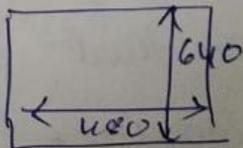
with 1600 x 1200.

$$f_v = \frac{96 \times 1000}{1200 \times 0.95} = 76 \text{ Hz}.$$

**Image Resolution** :- It refers to pixel spacing, the distance b/w two pixels.

**Screen Resolution** :- It refers to the no. of pixels in the horizontal & vertical direction on the screen.

**Screen Resolution with 640 x 480.**



Increasing the resolution, the image will become smaller due to screen displaying more pixels per inch.

13 inch	640 x 480
15 inch	800 x 600
17 inch	1024 x 768
21 inch	1024 x 768



# *Computer Graphics And Multimedia*

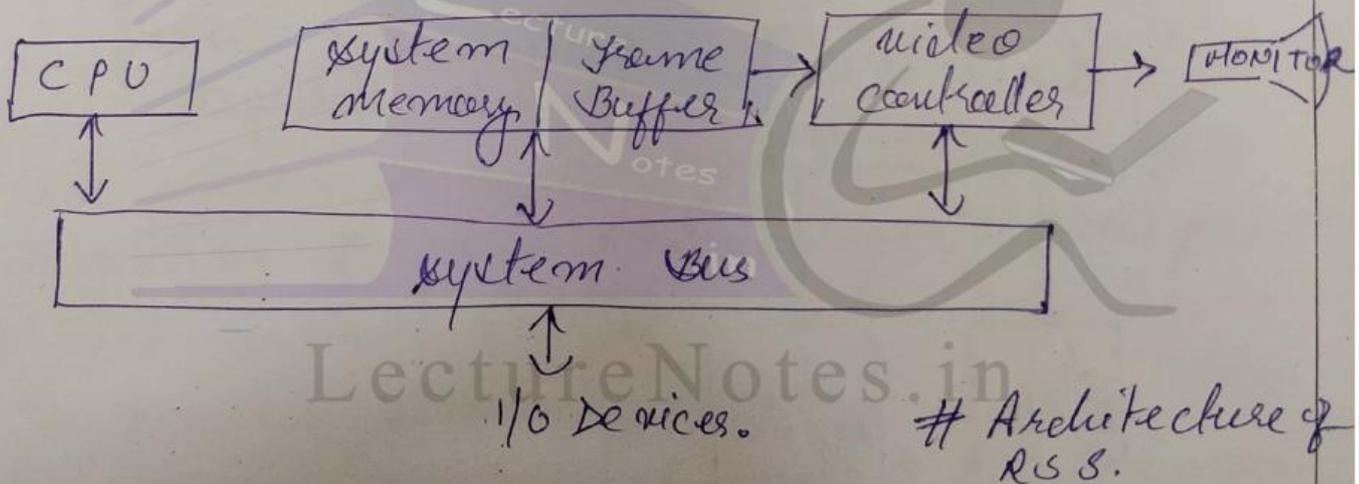
Topic:  
*Raster Scan Systems*

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# Raster Scan Systems

Raster scan systems :-

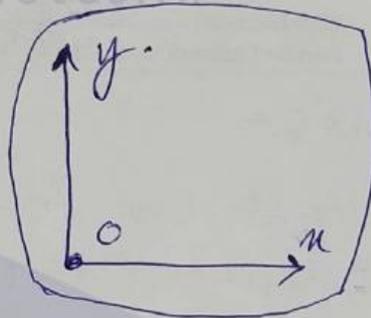
Interactive - allows user to interact with the graphical info presented on the display using some of the input devices.  
Eg:- video game controller.



# Architecture of RSS.

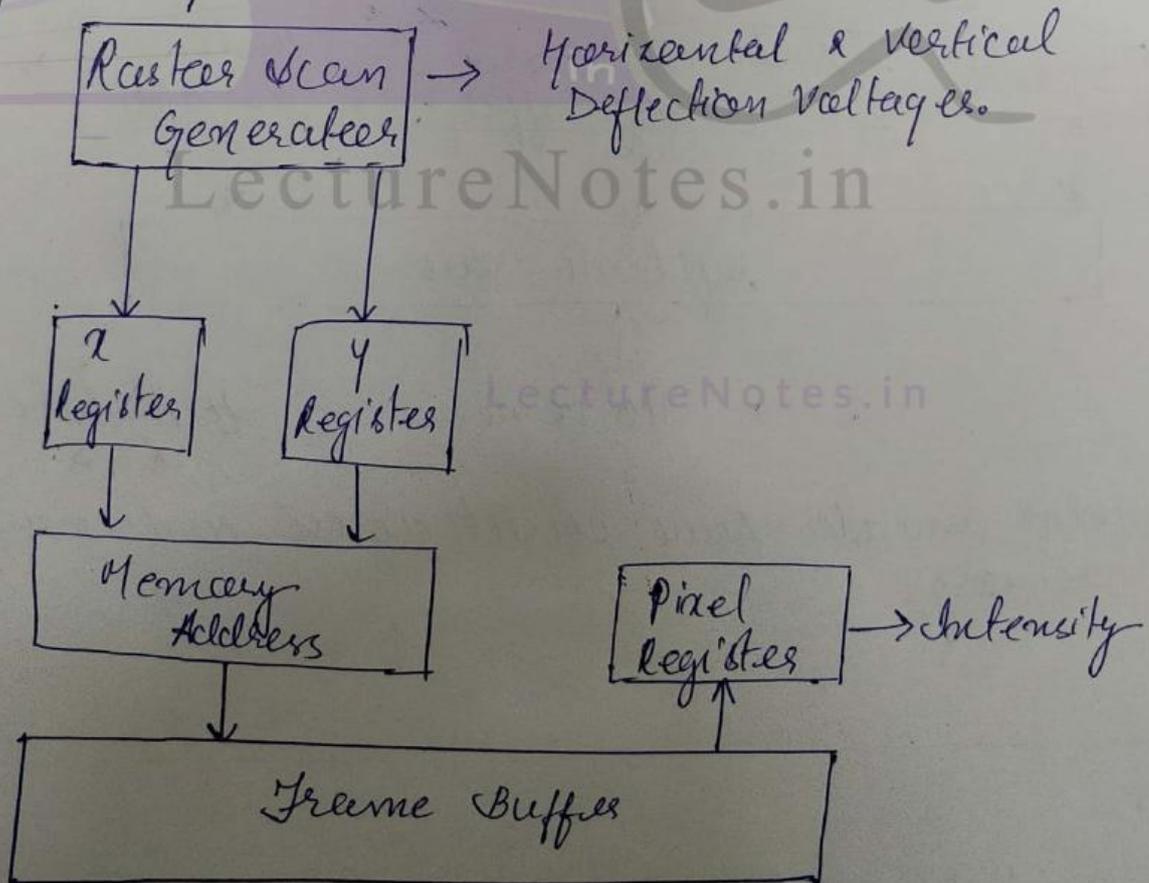
→ video controller have direct access to frame buffer memory.

Frame Buffer - Stores the screen position in form of pixels. And pixels are further stored as a cartesian co-ordinates. Origin of the screen is bottom-left corner.



Origin of the co-ordinate system.

Refresh Operations;



## Refresh Operations

Two registers are used to store co-ordinates  $x$  &  $y$  respectively.

$x$  register is set to 0

$y$  register is set to  $y_{max}$

The value stored in frame buffer for the pixel position  $x$  is retrieved & used to set the intensity of the CRT beam.

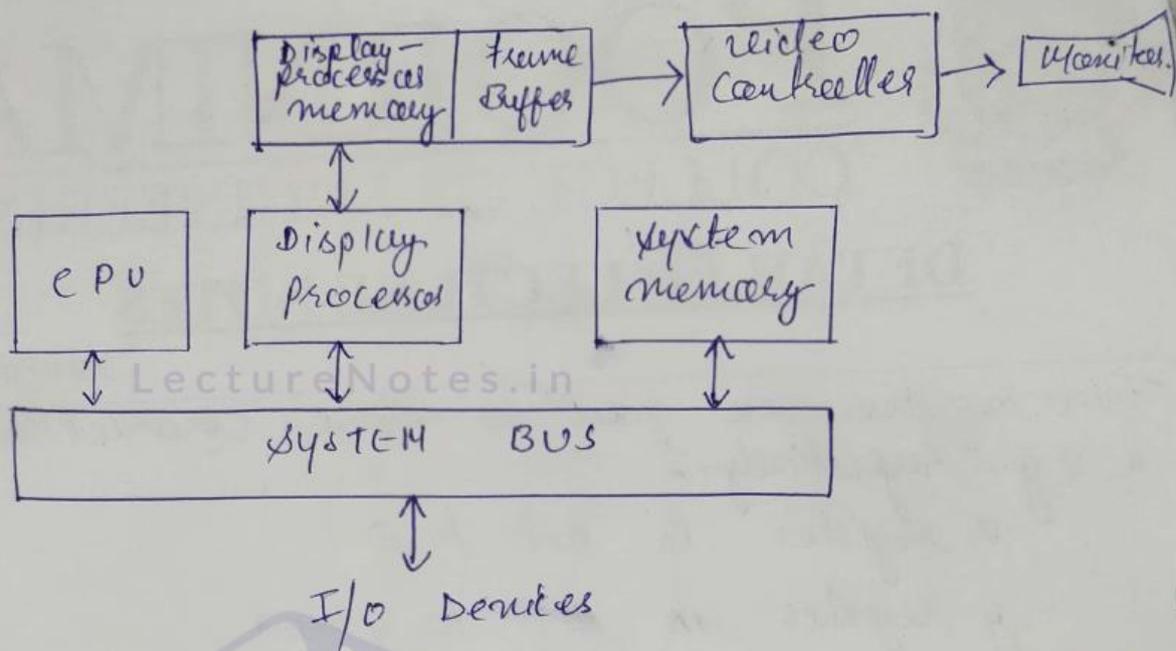
The  $x$  register is incremented by 1.

$y$  register is decremented by 1.

And process is repeated for each pixel. After completion of every pixel, registers are reset to their original values.

The process shown, generally, is very slow.

To achieve the refresh rate of 60 frames per second, video controllers retrieve multiple pixel values from refresh buffer. A separate register is used to store the intensity values.



Display processor - specialized I/O processor which is used to convert digital information from the CPU into the analog value by the display device. It will interpret a display list / frame buffer to produce picture and characters on the screen. which is known as scan conversion.

Display processor also performs various operations:-  
 generating lines styles  
 display color areas  
 perform transformation & manipulation



# *Computer Graphics And Multimedia*

Topic:

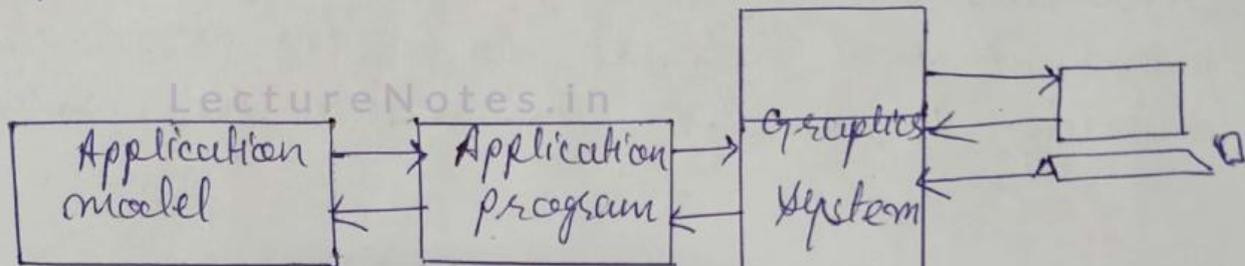
*Frame Work For Interactive Graphics*

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# Framework for Interactive Graphics

conceptual framework for interactive graphics:-



A computer receives i/p from interaction device & output image to a display device.

The framework has 3 components;

1. Application program; It stores the data, retrieves the data, it creates the data to be stored in application model. It handles user i/p.

2. Application model; It will store data as object to be pictured on the screen.

App program, on receiving i/p from user it produces views by sending it to the graphics system.

Graphics system, deals with the actual representation of the object on the screen, what to be appeared & how it should appear.

OS is intermediate b/w app program & display hardware.

The fundamental task of interactive system is to specify what data or objects are to be generated and how they will be present pictorially. and also, how the user & app program will interact in order to create & modify the model & its visual representation.

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# *Computer Graphics And Multimedia*

Topic:

*Scan Conversion Or Rsterrization*

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# Scan conversion or Rasterisation

Scan conversion :-

The process of representing continuous graphics objects as a collection of discrete pixels is called scan conversion.

It is the responsibility of graphics system as the app program to convert each primitive from its geometric definition into a set of pixels.

This conversion task is called scan conversion or rasterisation.

Scan converting a point :-

The mathematical point  $(x, y)$  where  $x$  &  $y$  are real no. within screen area, needs to be scan converted to a pixel at a location  $(x' & y')$ .

$$x' = \text{floor}(x) \quad y' = \text{floor}(y)$$

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The origin is lower left corner of the grid.  
 points must satisfy following conditions:-

$$x' \leq x \leq x'+1 \quad \&$$

$$y' \leq y \leq y'+1$$

Eg:-  $P_1(1.7, 0.8)$        $P_2(2.2, 1.3)$   
 $P_3(2.8, 1.9)$

Scan conversion.

$P_1(1.7, 0.8)$

$P_2(2.2, 1.3)$

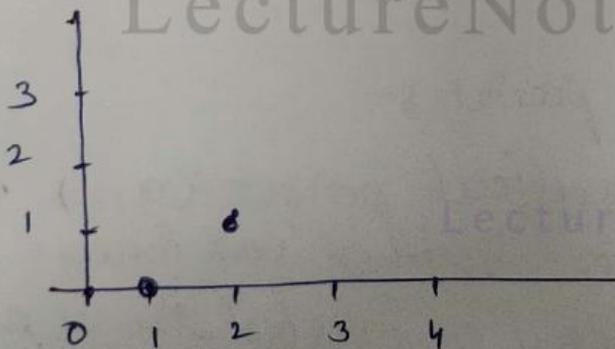
$P_3(2.8, 1.9)$

$\lceil \text{Floor}(x+1) \rceil$

$P_1(1, 0)$

$P_2(2, 1)$

$P_3(2, 1)$



# Second Approach;

$$x' = \text{Floor}(x + 0.5)$$

$$y' = \text{Floor}(y + 0.5)$$

# Digital Differential Analyzer (DDA) Line Algorithm

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Now for some example :-

$$P_1(1.7, 0.8)$$

$$P_2(2.2, 1.3)$$

$$P_3(2.8, 1.9)$$

$$y_{\text{coord}}(x+0.5); \text{ } \overline{f} = \text{localy } \overline{f} + \Delta y$$

$$P_1(2, 1)$$

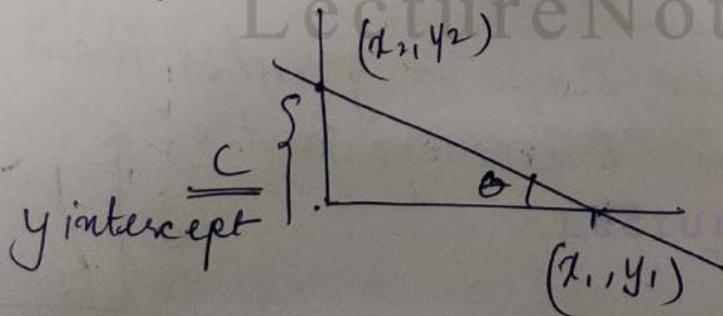
$$P_1(2, 1)$$

$$P_3(3, 2)$$

scan converts into line :-

DDA (Digital differential Analyzer) Algorithm.

$$y = mx + c$$



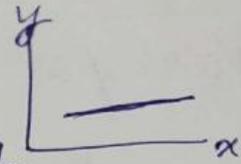
$$m - \text{slope} = \tan \theta = \frac{p}{B}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\Delta y}{\Delta x}$$

Now we are considering 3 cases;  
when slope is less than or equal to one  
which means,  $x$  co-ordinate is increasing  
faster than  $y$  co-ordinate.

So,  $x$  is always increasing by 1.



$$x_2 - x_1 = 1$$

$$\text{or, } x_{k+1} - x_k = 1$$

$$\text{or, } \Delta x = 1.$$

$$\text{now, } m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{1}$$

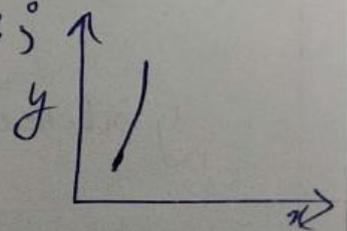
$$\text{or, } m = \frac{y_{k+1} - y_k}{1}$$

$$\boxed{y_{k+1} = y_k + m}$$

second case; when slope is greater than one.  
which means  $y$  is increasing faster;

$$\text{here, } y_2 - y_1 = 1$$

$$\Delta y = 1$$



$$\text{now, } m = \frac{\Delta y}{\Delta x} = \frac{1}{x_2 - x_1}$$

## DDA Line Drawing Example

$$m = \frac{1}{x_{k+1} - x_k}$$

$$x_{k+1} - x_k = \frac{1}{m}$$

$$x_{k+1} = \frac{1}{m} + x_k$$

Example :-

Suppose we have following co-ordinates, for the line :-  $P_1(2, 4)$  &  $P_2(5, 7)$ .

$$x_1 = 2, \quad x_2 = 5$$

$$y_1 = 4, \quad y_2 = 7$$

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{5 - 2} = \frac{3}{3} = 1$$

Case 1, here,  $x$  is increasing faster.

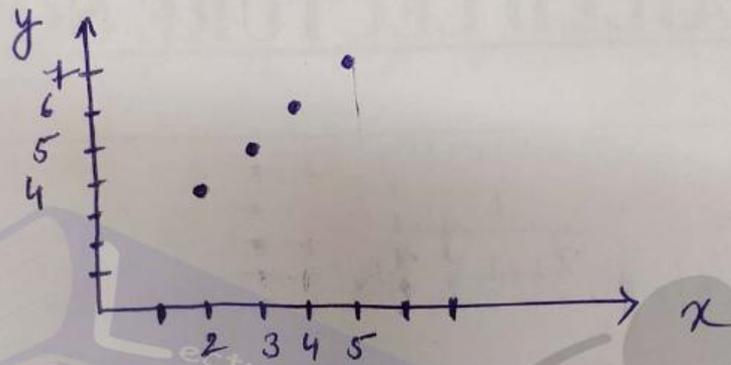
$$y_{k+1} = y_k + m = 4 + 1 = 5$$

$$y_{k+1} = y_k + m = 5 + 1 = 6$$

$$y_{k+1} = y_k + m = 6 + 1 = 7$$

Points for the pixels are:-

$P_1 (2, 4)$        $P_2 (3, 5)$        $P_3 (4, 6)$   
 $P_4 (5, 7)$ .

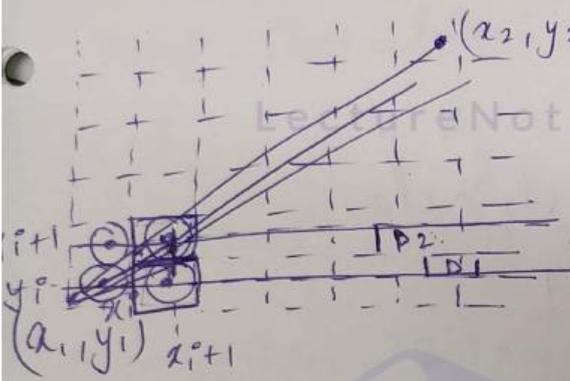


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# Bresenham's Line Drawing Algorithm

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## Bresenham's Line Algorithm :-



Assumptions before starting the derivation of algo :-

Assumption 1 - two endpoints  $(x_1, y_1)$  &  $(x_2, y_2)$

Assumption 2 - slope is  $< 45^\circ$   
 $0 \leq m \leq 1$

Assumption 3,  $x_1 < x_2$

Assumption 4,  $y = mx + b$

For,  $x_i + 1$

$$y \text{ for } x_i + 1 \text{ is :- } y = m(x_i + 1) + b$$

$$\& \ m = \frac{\Delta y}{\Delta x}$$

$$d_1 = y - y_i = m(x_i + 1) + b - y_i$$

Distance b/w  $y$  &  $y_i$

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$$d_2 = y_{i+1} - y = y_{i+1} - m(x_i + 1) + b$$

$$d_1 - d_2 = m(x_i + 1) + b - y_i - (y_{i+1} - m(x_i + 1))$$

$$= 2m(x_i + 1) - 2y_i + 2b - 1$$

This difference will help us to know which pixel is nearer to the line.

If ;  $d_1 - d_2 < 0$  ; i.e  $d_1 < d_2$

choose pixel,  $y_i$

$$\text{So, } y_{i+1} = y_i$$

If ;  $d_1 - d_2 > 0$  ; i.e  $d_1 > d_2$

choose pixel,  $y_{i+1}$

$$\text{So, } y_{i+1} = y_i + 1$$

$P_i = -\Delta x (d_1 - d_2)$  ; multiply with  $\Delta x$  to avoid fractional calculations.

$$= (2m(x_i + 1) - 2y_i + 2b - 1) \times \Delta x$$

$$\underline{\Delta x} \left( 2 \times \frac{\Delta y}{\Delta x} (x_i + 1) - 2y_i + 2b - 1 \right)$$

$$P_i = 2 \Delta y (x_i) - 2 \Delta x \cdot y_i + \underbrace{2 \Delta y + 2 \Delta x b - \Delta x}_{\rightarrow \text{constant}}$$

$$P_i = 2 \Delta y x_i - 2 \Delta x y_i + c$$

next iteration;

$$P_{i+1} = 2 \Delta y x_{i+1} - 2 \Delta x y_{i+1} + c$$

subtract;  $P_{i+1} - P_i$

$$= 2 \Delta y x_{i+1} - 2 \Delta x y_{i+1} + c - 2 \Delta y x_i + 2 \Delta x y_i - c$$

$$= 2 \Delta y (x_{i+1} - x_i) - 2 \Delta x (y_{i+1} - y_i)$$

$$= 2 \Delta y - 2 \Delta x (y_{i+1} - y_i)$$

$$P_{i+1} = P_i + 2 \Delta y - 2 \Delta x (y_{i+1} - y_i)$$

$$\text{If } P_i < 0 \quad P_{i+1} = P_i + 2 \Delta y$$

$$P_i > 0 \quad P_{i+1} = P_i + 2 \Delta y - 2 \Delta x$$

Initial parameter;

$$P_0 = 2 \Delta y - \Delta x$$

$$P_0 = 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + 2\Delta x b - \Delta x$$

$$b = y_0 - \left(\frac{\Delta y}{\Delta x}\right) x_0$$

$$= 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + 2\Delta x \times \left(y_0 - \left(\frac{\Delta y}{\Delta x}\right) x_0\right) - \Delta x$$

$$= 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + 2\Delta x y_0 -$$

$$2\Delta x \times \left(\frac{\Delta y}{\Delta x}\right) x_0 - \Delta x$$

$$= 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + 2\Delta x y_0 - 2\Delta y x_0 - \Delta x$$

$$= 2\Delta y - \Delta x$$

Example of Bresenham's Algo :-

$$(x_1, y_1) = (20, 10)$$

$$(x_2, y_2) = (30, 18)$$

$$m = 0.8$$

$$\Delta x = 10, \\ (30 - 20)$$

$$\Delta y = 8 \\ (18 - 10)$$

# Bresenham's Line Drawing Example 1

$$P_0 = 2Ay - \Delta x$$

$$= 2 \times 8 - 10 = 16 - 10 = 6.$$

$i$	$P_i$	$x_{i+1}, y_{i+1}$	$P_i > 0$ $y_{i+1} = y_i + 1$ $= y_i + 1 = 10 + 1 =$
0	6	21, 11	
1	2	22, 12	
2	-2	23, 12	
3	14	24, 13	
4	10	25, 14	
5	6	26, 15	
6	2	27, 16	
7	-2	28, 16	
8	14	29, 17	
9	10	30, 18	

## Bresenham's Line Drawing Example 2

Example :- Draw a line b/w (5, 12) & (15, 20) with BDDA.

Step 1 - calculate slope;

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8}{10} = 0.8 < 1$$

$$P_i = 2\Delta y - \Delta x$$

$$P_{i+1} = P_i + 2\Delta y - 2\Delta x(y_{i+1} - y_i)$$

$$P_i \geq 0$$

$$y_{i+1} = y_i + 1$$

$$P_i < 0$$

$$y_{i+1} = y_i$$

If  $m > 1$

$$P_i = 2\Delta x - \Delta y$$

$$P_{i+1} = P_i + 2\Delta x - 2\Delta y(x_{i+1} - x_i)$$

$$P_i \geq 0$$

$$x_{i+1} = x_i + 1$$

$$P_i < 0$$

$$x_{i+1} = x_i$$

$$P_0 = 2 \times 8 - 10 = 6$$

$$P_1 = P_0 + 2\Delta y - 2\Delta x(y_{i+1} - y_i)$$

$$6 + 2 \times 8 - 2 \times 10(13 - 12) = 6 + 16 - 20 = 2$$

$$P_2 = P_1 + 2 \times 8 - 2 \times 10(14 - 13) = 2 + 16 - 20 = -2$$

$$P_3 = P_2 + 2 \times 8 - 2 \times 10(14 - 14) = -2 + 16 - 20 \times 0 = 14$$

$$P_4 = P_3 + 2 \times 8 - 2 \times 10(15 - 14) = 14 + 16 - 20 = 10$$

$$P_5 = P_4 + 2 \times 8 - 2 \times 10(16 - 15) = 10 + 2 \times 8 - 20 = 6$$

$$P_6 = P_5 + 16 - 20(17 - 16) = 6 + 16 - 20 = 2$$

$$P_7 = 2 + 16 - 20(18 - 17) = -2 \quad \Bigg| \quad P_8 = 14 + 16 - 20(19 - 18)$$

$$P_8 = -2 + 16 - 20(0) = 14 \quad \Bigg| \quad 14 + 16 - 20 = 10$$

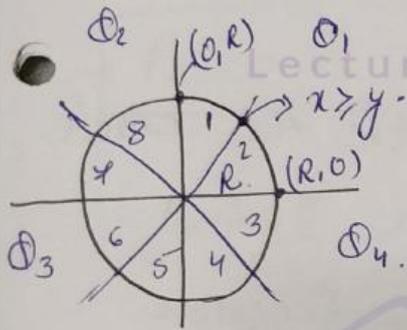
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Tables for the points.

$n_c$	$x_i$	$y_i$	$p_i$	$x_{i+1}$	$y_{i+1}$
0	<u>5</u>	<u>12</u>	6	6	13
1	6	13	2	7	14
2	7	14	-2	8	14
3	8	14	14	9	15
4	9	15	10	10	16
5	10	16	6	11	17
6	11	17	2	12	18
7	12	18	-2	13	18
8	13	18	14	14	19
9	14	19	10	<u>15</u>	<u>20</u>
10					
11					

# Mid Point Circle Drawing Algorithm

Mid Point circle Algo:-



circle is having 8 fold symmetry property.

Divide the circle into 8 octets.

Equation of circle:-

$$x^2 + y^2 - R^2 = 0$$

Here,  $x$  - increase unit interval.

$y$  - can increase/decrease.

Next points can be:-

$$(x_k + 1, y_k) \text{ or } (x_k + 1, y_k - 1)$$

which point to choose depends on decision parameter.

$$\begin{aligned} \text{Mid Point is:- } & \frac{x_k + 1 + x_k + 1}{2}, \frac{y_k + y_k - 1}{2} \\ & = x_k + 1, y_k - \frac{1}{2} \end{aligned}$$

Page No.:

$$P_k = f(x_{k+1}, y_{k-\frac{1}{2}})$$

$$f(x, y) = x^2 + y^2 - R^2$$

put the value of  $P_k$  in  $f(x, y)$

$$(x_{k+1})^2 + (y_{k-\frac{1}{2}})^2 - R^2 = 0$$

$P_k < 0$ , mid point is inside the circle  
 $y_k$  is closer.

if  $P_k \geq 0$ , mid point is outside the circle  
 $y_{k-1}$  is closer.

Similarly, we need successive decision parameters.

$$P_{k+1} = f(x_{k+1} + 1, y_{k+1} - \frac{1}{2})$$

$$= (x_{k+1} + 1)^2 + (y_{k+1} - \frac{1}{2})^2 - R^2$$

$$P_{k+1} - P_k = (x_{k+1} + 1)^2 + (y_{k+1} - \frac{1}{2})^2 - R^2 -$$

$$(x_k + 1)^2 - (y_k - \frac{1}{2})^2 + R^2$$

$$((x_k + 1) + 1)^2 + (y_{k+1} - \frac{1}{2})^2 - (x_k + 1)^2 - (y_k - \frac{1}{2})^2$$

$$(x_k + 1)^2 + 1 + 2(x_k + 1) + (y_{k+1})^2 + \frac{1}{4} - 2 \times \frac{1}{2} (y_{k+1})$$

$$- (x_k + 1)^2 - (y_k)^2 - \frac{1}{4} + y_k$$

$$P_{k+1} - P_k = 2(x_{k+1}) + (y_{k+1})^2 - (y_k)^2 - y_{k+1} + y_k + 1$$

$$= 2(x_k + 1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

$$P_{k+1} = P_k + 2(x_k + 1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

Initial decision parameters:-

$$x_k = 0, \quad y_k = 2$$

$$P_k = (x_k + 1)^2 + (y_k - 1/2)^2 - 2^2$$

$$(0 + 1)^2 + (2 - 1/2)^2 - 2^2$$

$$1 + 2^2 + \frac{1}{4} - 2 \times 1 \times 2 - 4^2$$

$$1 + \frac{1-2}{4} = \frac{5-2}{4}$$

$$P_0 = \frac{5}{4} - 2$$

$$\text{or, } \frac{1.25 - 2}{1}$$

$$\underline{\underline{1 - 2}}$$

## Mid Point Circle Drawing Example

$$P_k \geq 0, \quad NC = y_{k+1} = y_k - 1 \\ (x_{k+1}, y_k - 1)$$

$$P_k < 0, \quad NC = y_{k+1} = y_k \\ (x_{k+1}, y_k)$$

Example 8

$$P_k = 1 - r.$$

$$P_{k+1} = P_k + 2(x_{k+1}) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

Given radius = 8.

$$P_0 = 1 - r = 1 - 8 = -7$$

$$P_{k+1} = -7 + 2(0+1) + (8^2 - 8^2) - (8-8) + 1 \\ -7 + 2 + 0 + 0 + 1 = -7 + 3 = -4$$

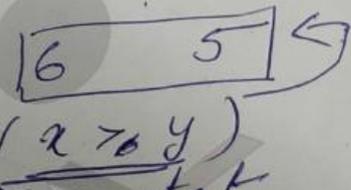
$$P_2 = -4 + 2(2+1) + 0 - 0 + 1 \\ -4 + 4 + 1 = 1$$

$$P_3 = 1 + 2(3+1) + (49 - 64) - (7-8) + 1 \\ 1 + 6 - 15 + 1 + 1 = -6$$

$$P_4 = -6 + 2(4+1) + (36 - 49) - (6-7) + 1 \\ -6 + 8 - 13 + 1 + 1 = -3$$

$$P_5 = 3 + 2(5+1) + (36 - 49) - (6-7) + 1 \\ 3 + 10 - 13 + 1 + 1 = 2$$

k iteration	$x_k$	$y_k$	$P_k$	$x_{k+1}$	$y_{k+1}$
0	0	8	-7	1	8
1	1	8	-4	2	8
2	2	8	1	3	7
3	3	7	-6	4	7
4	4	7	3	5	6
5	5	6	2	6	5



whenever,  $x > y$ , end of 1st octet.

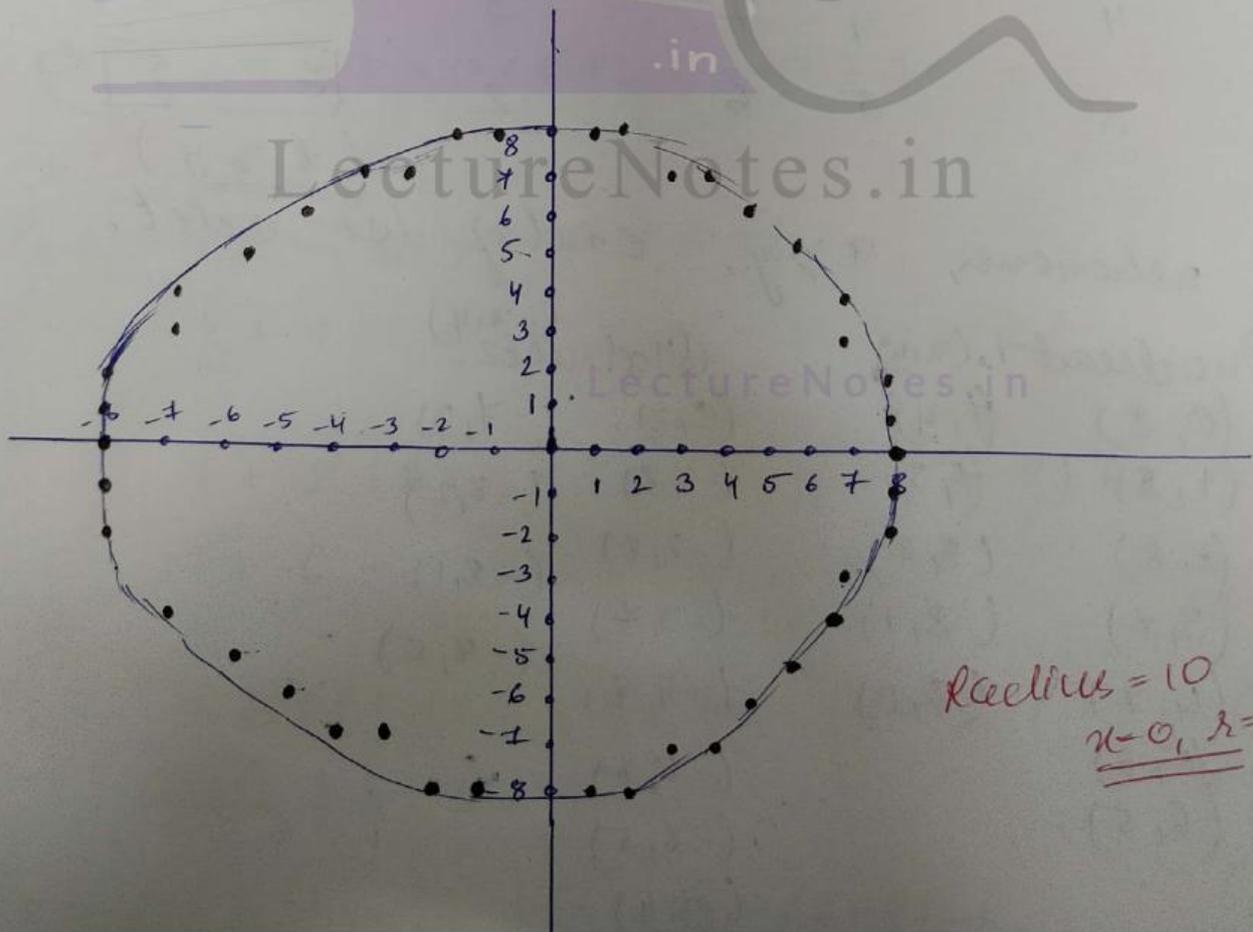
- Quadrant 1.  $(x, y)$
- (0, 8)
  - (1, 8)
  - (2, 8)
  - (3, 7)
  - (4, 7)
  - (5, 6)
  - (6, 5)
- Quadrant 2.  $(-x, y)$
- (-7, 3)
  - (-8, 2)
  - (-8, 1)
  - (-8, 0)
  - (-4, 7)
  - (-5, 6)
  - (-6, 5)
  - (-7, 4)

Quadrant 3  $(-x, -y)$

- $(0, 8)$
- $(-1, -8)$
- $(-2, -8)$
- $(-3, -7)$
- $(-4, -7)$
- $(-5, -6)$
- $(-6, -5)$
- $(-7, -4)$
- $(-7, -3)$
- $(-8, -2)$
- $(-8, -1)$
- $(-8, 0)$

Quadrant 4  $(x, -y)$

- $(0, 8)$
- $(1, -8)$
- $(2, -8)$
- $(3, -7)$
- $(4, -7)$
- $(5, -6)$
- $(6, -5)$
- $(7, -4)$
- $(7, -3)$
- $(8, -2)$
- $(8, -1)$
- $(8, 0)$

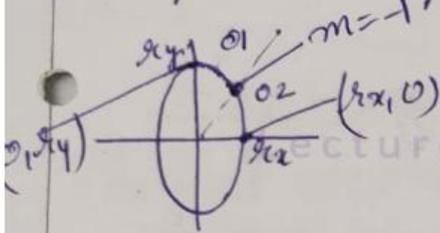


radius = 10  
 $x=0, x=y$

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# Mid Point Ellipse Drawing Algorithm

Midpoint Ellipse Drawing Algorithm :-



Follow 2 different procedures :-

o1, slope,  $m < 1$

o2, slope,  $m > 1$

If  $m < 1$ ,  $x$  moves unit interval

$y_{i+1} ?$ ,  $y_i$ ,  $y_{i-1}$

NC  $\rightarrow (x_{i+1}, y_i)$   $(x_{i+1}, y_{i-1})$

If  $m > 1$ ,  $y$  moves unit interval

$x_{i+1} ?$ ,  $x_i$ ,  $x_{i+1}$

NC  $\rightarrow (x_i, y_{i+1})$   $(x_{i+1}, y_{i-1})$

Now where to stop octet 1 & start octet 2

Eq<sup>n</sup> of ellipse :-

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$x^2 y^2 + y^2 x^2 - x^2 y^2 = 0 \quad \text{--- (1)}$$

$$m = \frac{dy}{dx}$$

$$2x y^2 \cdot x \geq 2 x^2 y \cdot y$$

LectureNotes.in

Octet.  $y$   $m < 1$

Expected points,  $(x_{k+1}, y_k)$   $(x_{k+1}, y_{k-1})$

mid point is :-

$$\frac{x_k + x_{k+1}}{2}, \frac{y_k + y_{k-1}}{2}$$

$$= \frac{x_{k+1}}{2}, y_{k-1/2} \rightarrow \text{put in eq. 1.}$$

Decision parameters :-

$$P_{1k} = (x_{k+1})^2 y^2 + (y_{k-1/2})^2 x^2 - x^2 y^2 = 0$$

$$P_{1k+1} = \frac{(x_{k+1} + 1)^2 y^2 + (y_{k+1} - 1/2)^2 x^2 - x^2 y^2 = 0}{\rightarrow x_{k+1} + 1}$$

$$(x_{k+1} + 2)^2 y^2 + (y_{k+1} - 1/2)^2 x^2 - x^2 y^2 = 0$$

$$P_{1k+1} - P_{1k} = \frac{(x_{k+1} + 1)^2 y^2 + y^2 + 2(x_{k+1} + 1)y^2 + y_{k+1}^2 x^2}{+ 1/4 x^2} - y_{k+1}^2 x^2 - x^2 y^2 - (x_{k+1})^2 y^2 -$$

$$y_k^2 x^2 - 1/4 x^2 + y_k x^2 + x^2 y^2$$

$$P_{ik+1} = P_{ik} + r_y^2 + 2(x_{k+1})r_y^2 + r_x^2(y_{k+1}^2 - y_k^2)$$

$$- r_x^2(y_{k+1} - y_k) \rightarrow \text{Decision parameters for 1st octet.}$$

LectureNotes.in

Initial decision parameters:-  
 substitute  $(0, r_y)$  in  $P_{ik}$

$$P_{ik} = (0+1)^2 r_y^2 + (r_y - 1/2)^2 r_x^2 - r_x^2 r_y^2$$

$$= r_y^2 + r_y^2 r_x^2 + \frac{r_x^2}{4} - r_y r_x^2 - r_x^2 r_y^2$$

$$P_0 = r_y^2 + \frac{r_x^2}{4} - r_y r_x^2$$

Step taken,  $2r_y^2 x \geq 2r_x^2 y$

$$P_{ik} \geq 0, \quad (x_{k+1}, y_{k-1})$$

$$P_{ik} < 0 \quad (x_{k+1}, y_k)$$

Octet 2 :-  $m > 1,$

Expected points :-  $(x_k, y_{k-1})$

$(x_{k+1}, y_{k-1})$

midpoint :-  $(x_k + 1/2, y_k - 1)$

put this in eq 1.

$$P_{2k} = (x_k + 1/2)^2 \sigma_y^2 + (y_k - 1)^2 \sigma_x^2 - \sigma_x^2 \sigma_y^2 = 0$$

$$x_k^2 \sigma_y^2 + \frac{1}{4} \sigma_y^2$$

$$P_{2k+1} = (x_{k+1} + 1/2)^2 \sigma_y^2 + (y_{k+1} - 1)^2 \sigma_x^2 - \sigma_x^2 \sigma_y^2 = 0$$

$$(x_{k+1} + 1/2)^2 \sigma_y^2 + (y_k - 1)^2 \sigma_x^2 - \sigma_x^2 \sigma_y^2 = 0$$

$$P_{2k+1} - P_{2k} = x_{k+1}^2 \sigma_y^2 + \frac{\sigma_y^2}{4} + x_{k+1} \sigma_y^2 + (y_k - 1)^2 \sigma_x^2 +$$

$$\sigma_x^2 - 2(y_k - 1) \sigma_x^2 - \sigma_x^2 \sigma_y^2 - x_k^2 \sigma_y^2$$

$$- \frac{\sigma_y^2}{4} - x_k \sigma_y^2 - (y_k - 1)^2 \sigma_x^2 + \sigma_x^2 \sigma_y^2$$

$$P_{2k+1} = P_{2k} + \sigma_x^2 - 2\sigma_x^2 (y_k - 1) + \sigma_y^2 (x_{k+1}^2 - x_k^2) + \sigma_y^2 (x_{k+1} - x_k)$$

can't derive optimal decision parameters.

$$\text{if } P_{2k} \geq 0, \quad (x_k, y_k - 1)$$

$$P_{2k} < 0, \quad (x_{k+1}, y_k - 1)$$

## Mid Point Ellipse Drawing Example

Example :-

Region 1 :-

$$P_{1k} = x_k^2 y_k + \frac{x_k^2}{4} - y_k x_k^2$$

$$P_{1k+1} = P_{1k} + x_k^2 y_k + 2(x_{k+1}) x_k^2 y_k + x_k^2 (y_{k+1}^2 - y_k^2) - x_k^2 (y_{k+1} - y_k)$$

$$P_{1k} \geq 0 \Rightarrow (x_{k+1}, y_k - 1)$$

$$P_{1k} < 0 \Rightarrow (x_{k+1}, y_k)$$

Region 2 :-

$$P_{2k} = (x_{k+1/2})^2 x_k^2 + (y_k - 1)^2 x_k^2 - x_k^2 x_k^2$$

$$P_{2k+1} = P_{2k} + x_k^2 x_k - 2x_k^2 x_k (y_k - 1) + x_k^2 y_k (x_{k+1}^2 - x_k^2) + x_k^2 y_k (x_{k+1} - x_k)$$

$$P_{2k} \geq 0 \Rightarrow (x_k, y_k - 1)$$

$$P_{2k} < 0 \Rightarrow (x_{k+1}, y_k - 1)$$

$$\mu_x = 8, \mu_y = 6.$$

Region 1.

$$P_{10} = -332 < 0.$$

k	$x_k, y_k$	$P_{1k}$	$x_{k+1}, y_{k+1}$	$2x_{k+1}k_j^2$	$2y_{k+1}k_j^2$
0	0, 6	-332	1, 6	72	768
1	1, 6	-224	2, 6	144	768
2	2, 6	-44	3, 6	216	768
3	3, 6	208	4, 5	288	640
4	4, 5	-108	5, 5	360	640
5	5, 5	288	6, 4	432	512
6	6, 4	244	7, 3	504	384

stop

Region 2

$$P_{20} = -23 < 0$$

7	7, 3	-23	8, 2		
8	8, 2	361	8, 1		
9	8, 1	297	8, 0		



# *Computer Graphics And Multimedia*

Topic:

*Aliasing And Antialiasing*

Contributed By:

*Ankita Jiyani*

# Aliasing and Antialiasing

In computer graphics, antialiasing is a  $\text{CPU}$  technique for diminishing jaggies, stair-step like lines that should be smooth.

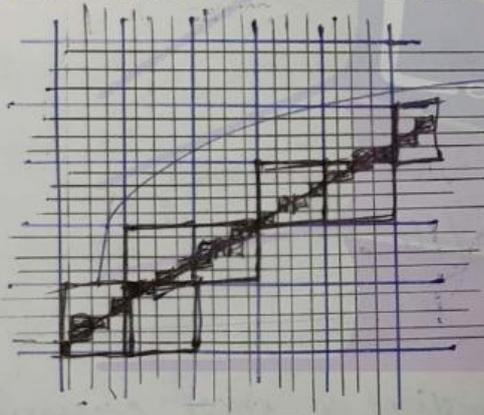
Jaggies occur because the  $\text{CPU}$  device the monitor or printer doesn't have enough resolution to represent a smooth line.

Antialiasing reduces the prominence of jaggies by surrounding the stairsteps with intermediate shades of gray, or colors. It reduces the jagged appearance of the lines but also makes them fuzzier.

Another method for reducing jaggies is a smoothing, in which the printer changes the size & horizontal alignment of dots to make curves smoother.

## Aliasing & Anti Aliasing.

When scan converting the pictures, jaggies appear on the screen, is k/a aliasing. The digital information from the CPU is sampled at low frequency due to which this phenomenon occurs a visible on the screen.



→ Jaggies

↳ No smooth edges.

↳ e.g. - playing high graphics game at low resolution.

The application or techniques which removes aliasing is referred as anti aliasing.

# Antialiasing Techniques

Anti Aliasing techniques :-

1. Super Sampling or Post filtering
2. Area Sampling or pre filtering.

Super sampling deals with fine grid. we are increasing the frequency of the sampling.

It will smooth the line, remove some of the jaggies.

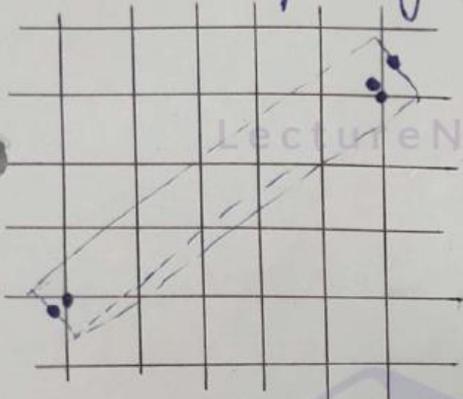
The line passes through twice as many columns, but has twice many jags, the jag is half as the previous one.

Disadvantages :-

- ↳ It requires more cost for memory
- ↳ more memory bandwidth.
- ↳ Scan conversion time.
- ↳ Higher resolution is expensive & not completely removing the problem of jags.

# Area Sampling

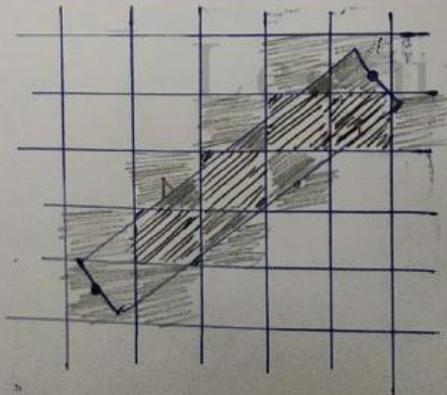
Area Sampling :-



we consider the line as a rectangle of required thickness covering a portion of a grid.

we assume a line contributes to each pixel's intensity an amount proportional to the percentage of the pixel's tile it covers.

Fully covered pixel colored black  
partially covered pixel colored gray



This blurring makes a line look better at a distance.

This technique is k/a unweighted area sampling.



# *Computer Graphics And Multimedia*

Topic:

*Basic Transformations In Graphics*

Contributed By:

*Ankita Jiyani*

# Transformations in Graphics

Basic Transformations :-

Translation :-

It is applied to the object by repositioning it along a straight line path from one coordinate location to other.

A 2D object can be translated by adding a translation distances,  $t_x$  &  $t_y$  to other position.

$$x' = t_x + x, \quad y' = t_y + y.$$

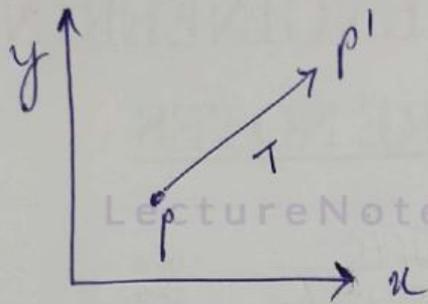
$t_x, t_y \rightarrow$  translation vectors or shift-vectors.

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad T = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

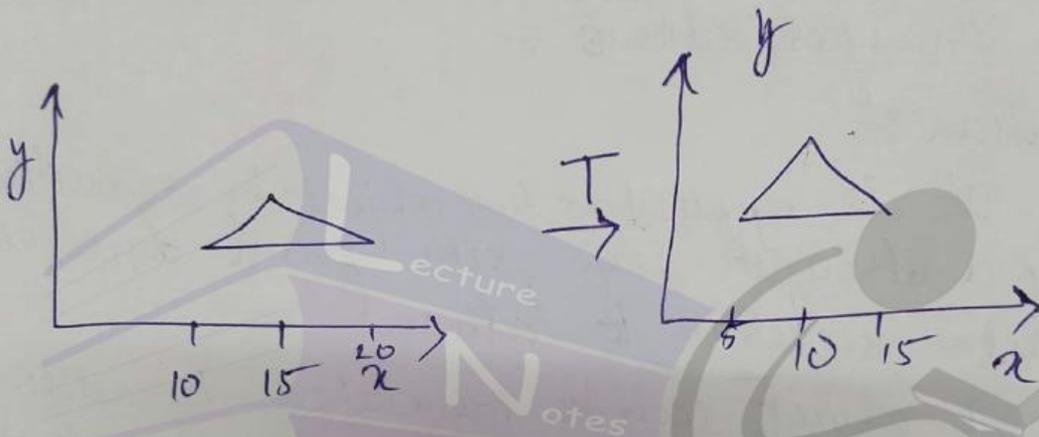
$$P' = P + T$$

Translation is rigid body transformation, that means the object without deformation.

Every point is translated by the same



A line is translated point



If we want to translate a circle, then we translate the center co-ordinates & redraw the figure in new location.

Rotation:-

It is applied along a circular path in an xy plane.

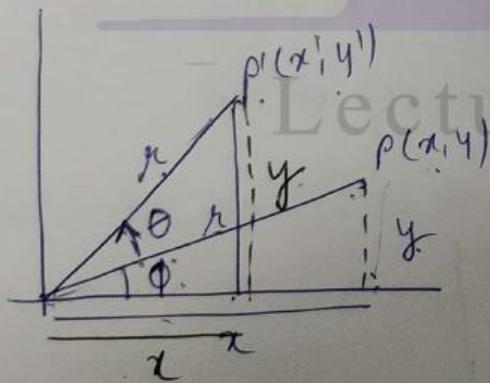
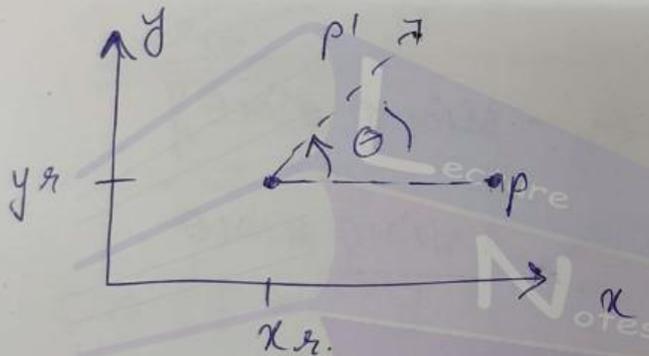
For rotation, we specify an angle  $\theta$  & a rotation center.

$a$ , position of rotation point  $(x_1, y_1)$

# Rotation in Graphics

Also k/a pivot point, about which the object is supposed to be rotated.

- positive values - counter clockwise direction
- negative values - clockwise direction.



$$\cos \phi = \frac{x}{r} = \frac{B}{H}$$

$$x = r \cos \phi$$

$$\sin \phi = \frac{y}{r} = \frac{P}{H}$$

$$y = r \sin \phi$$

Now Angel is :-

$$\underline{\underline{\phi + \theta}}$$

P to P'

$$\cos(\phi + \theta) = \frac{x'}{r}$$

$$\sin(\phi + \theta) = \frac{y'}{r}$$

$$\cos(A+B)$$

$$= \cos A \cos B - \sin A \sin B$$

$$\sin(A+B)$$

$$= \sin A \cos B + \cos A \sin B$$

$$x' = r \cos(\phi + \theta)$$

$$y' = r \sin(\phi + \theta)$$

$$x' = r [\cos \phi \cos \theta - \sin \phi \sin \theta]$$

$$\frac{r \cos \phi \cos \theta}{r} - \frac{r \sin \phi \sin \theta}{r}$$

$$y' = r [\sin \phi \cos \theta + \cos \phi \sin \theta]$$

$$= r \sin \phi \cos \theta + r \cos \phi \sin \theta$$

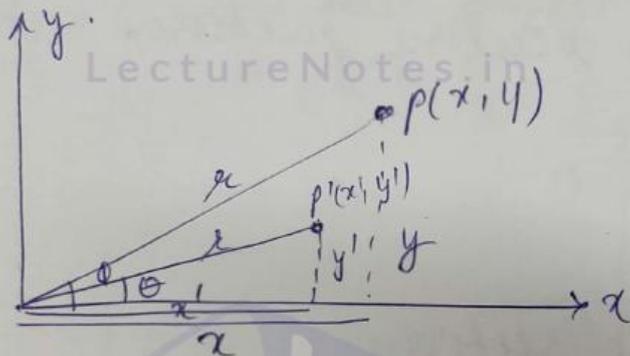
$$\boxed{\begin{aligned} x' &= x \cos \theta - y \sin \theta \\ y' &= x \sin \theta + y \cos \theta \end{aligned}}$$

$$P' = R \cdot P$$

$$\text{where } R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

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$$x' y' = [x \quad y] \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$



clockwise rotation

New Angle  $\rightarrow \phi - \theta$

$$\cos(\phi - \theta) = \frac{x'}{r}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\sin(\phi - \theta) = \frac{y'}{r}$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$x' = r \cos \phi \cos \theta + r \sin \phi \sin \theta$$

$$x' = x \cos \theta + y \sin \theta$$

$$y' = r \sin \phi \cos \theta - r \cos \phi \sin \theta$$

$$y' = y \cos \theta - x \sin \theta$$

$$P' = R \cdot P$$

$$[x' \ y'] = [x \ y] \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

### Scaling :-

It alters the size of an object. It is usually carried out by multiplying the vertex values by scaling factors.

$$x' = x \cdot S_x$$

$$y' = y \cdot S_y$$

$S_x$  - scales in x direction.

$S_y$  - scales in y direction.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = S \cdot P$$

$S_x, S_y < 1 \rightarrow$  Reduce the size of object

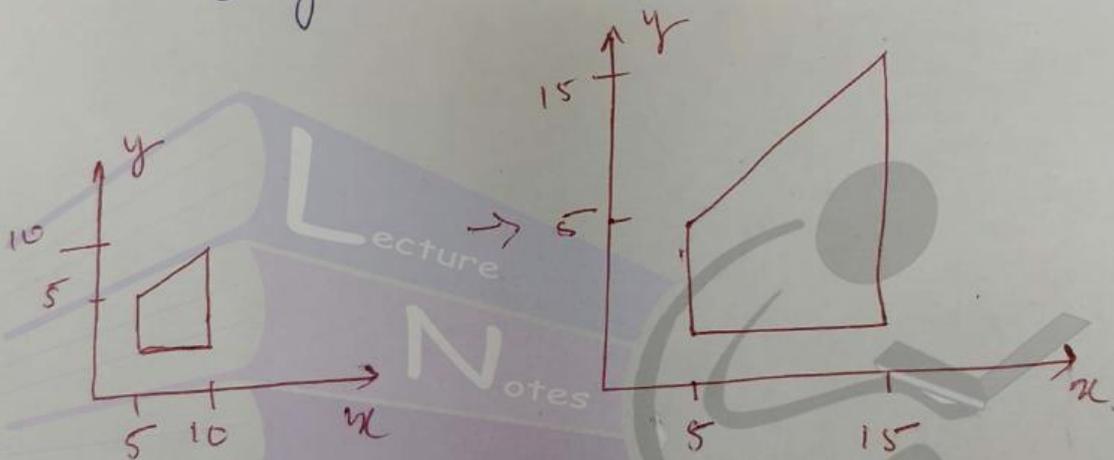
$S_x, S_y > 1 \rightarrow$  Increase the size of the object

$S_x, S_y = 1 \rightarrow$  Uniform scaling

# Scaling in Graphics

by,  $k_x < 1$ , moves object closer to the coordinate origin

$k_x, k_y > 1$ , moves object further from the origin.



LectureNotes.in

LectureNotes.in



# *Computer Graphics And Multimedia*

Topic:

*Homogeneous Matrix Representation*

Contributed By:

*Ankita Jiyan*

# Homogenous Matrix Representation

while designing the application, we perform translation, rotation, scaling and other transformations to fit the picture components into their proper position.

So, we are constructing the matrices, so that these transformation sequence can be efficiently processed.

Each transformation can be expressed in the general form as:-

$$P' = M_1 \cdot P + M_2 \quad \text{--- (1)}$$

where  $P$  &  $P'$  are column vectors

$M_1 \rightarrow 2 \times 2$  matrix containing multiplicative factors.

$M_2 \rightarrow 2$  element column matrix containing translational terms.

For translation

$M_1 \rightarrow$  identity matrix

For Rotation/Scaling.

$M_2 \rightarrow$  contains translational terms.

associated with pivot point as scaling point.

Suppose, we want to perform a series of transformations, i.e. scaling, rotating, translations etc. So an efficient way to do this task is to combine the transformations so that final co-ordinate positions are obtained directly from the initial co-ordinates, & eliminating the calculation of intermediate co-ordinate values.

$\rightarrow$  combine multiplication & translational terms for 2D geometric transformation into a single matrix representation by expanding  $2 \times 2$  matrix into  $3 \times 3$  matrix.

$\rightarrow$  To represent any 2D transformation as a matrix multiplication, we represent each cartesian co-ordinate with homogeneous co-ordinate triple  $(x_h, y_h, h)$ .

where,

$$x = \frac{xh}{h} \quad , \quad y = \frac{yh}{h}$$

Homogeneous co-ordinate representation  $\rightarrow$   
 $(hx, hy, h)$

$h \rightarrow$  homogeneous parameter  $h$   
 $\rightarrow$  non zero value.

There can be infinite no. of equivalent homogeneous representations for each coordinate point  $(x, y)$ .

$\rightarrow$  easy choice  $\Rightarrow h=1$ .

$\rightarrow$  each point is represented as  $\rightarrow (x, y, 1)$ .

$\rightarrow$  we are representing co-ordinates as homogeneous co-ordinates because,  $h \neq 0$  implementation for multiplication matrix is easy.

$\rightarrow$  Homogeneous coordinates are invented because of vanishing point. Projective space can't be efficiently represented by cartesian  $x-y$  coordinate system.

# Matrix Representation of Transformations

Representation of transformations in homogeneous coordinates.

① Translation :-

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = T(t_x, t_y) \cdot P$$

② Rotation :-

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = R(\theta) \cdot P$$

③ Scaling :-

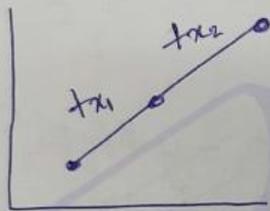
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Composite Transformations

composite transformation :-

Two successive of any transformation, by concatenation, or composition.

① Translation :-  
two & more than two. translation.



$$P' = T \cdot P.$$

$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = T_1 \cdot T_2 \cdot P$$

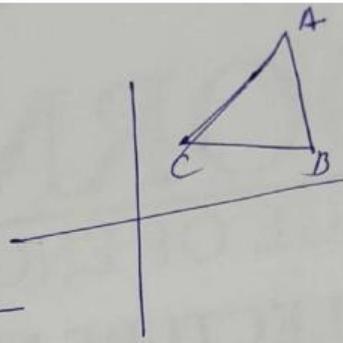
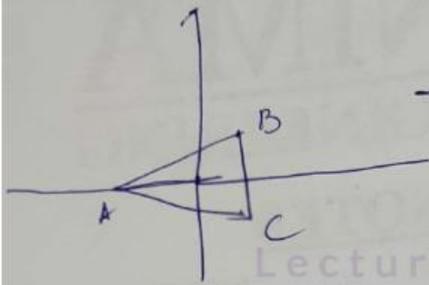
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{x1} \\ 0 & 1 & t_{y1} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_{x2} \\ 0 & 1 & t_{y2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & t_{x1} + t_{x2} \\ 0 & 1 & t_{y1} + t_{y2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x + t_{x1} + t_{x2} \\ y + t_{y1} + t_{y2} \\ 1 \end{bmatrix}$$

Two successive translation are additive.

② Reflection



$$P' = RP$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$P' = R_1 R_2 P$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \cos\theta_2 - y \sin\theta_2 \\ x \sin\theta_2 + y \cos\theta_2 \\ 1 \end{bmatrix}$$

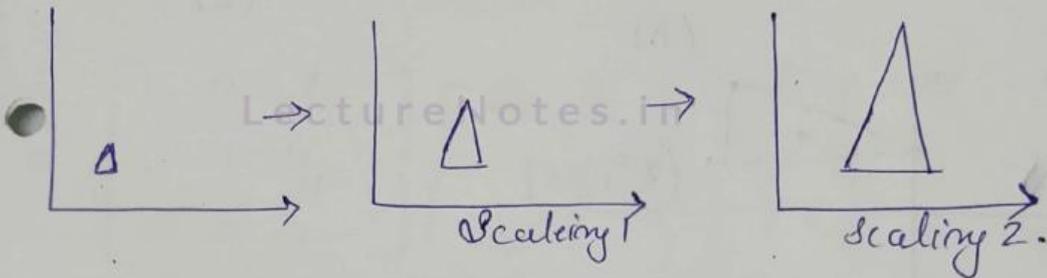
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 (x \cos\theta_2 - y \sin\theta_2) + (-\sin\theta_1) (x \sin\theta_2 + y \cos\theta_2) \\ \sin\theta_1 (x \cos\theta_2 - y \sin\theta_2) + \cos\theta_1 (x \sin\theta_2 + y \cos\theta_2) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Two successive rotations are additive

Scaling :-

$$P1 = S P.$$



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx_1 & 0 & 0 \\ 0 & Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx_1 & 0 & 0 \\ 0 & Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Sx_2 & 0 & 0 \\ 0 & Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx_1 \cdot Sx_2 & 0 & 0 \\ 0 & Sy_1 \cdot Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x \cdot Sx_1 \cdot Sx_2 \\ y \cdot Sy_1 \cdot Sy_2 \\ 1 \end{bmatrix}$$

Two successive scaling are multiplicative.

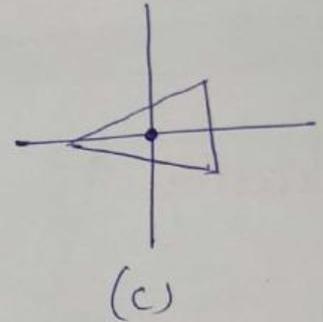
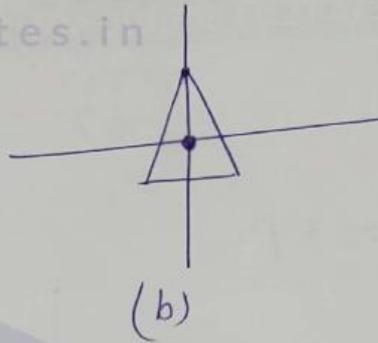
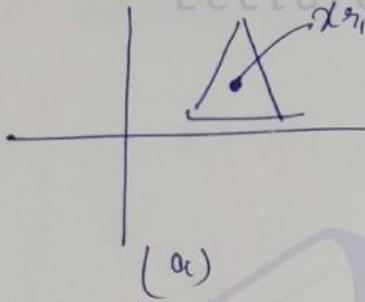
## Pivot-Point Rotation:-

① Translate

② Rotate

③ Translate

If above series of transformations performed in sequence then.



$$\begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_1 \\ 0 & 1 & -y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & x_1(1 - \cos \theta) + y_1 \sin \theta \\ \sin \theta & \cos \theta & y_1(1 - \cos \theta) + x_1 \sin \theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(x_1, y_1) \cdot R(\theta) \cdot T(-x_1, -y_1) = R(x_1, y_1, \theta)$$

# Fixed Point Scaling

PAGE NO. ....

Fixed point scaling :-

- ① Translate      ② Scaling.      ③ Inverse translate.

LectureNotes.in

$$= \begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_x & 0 & x_f(1-s_x) \\ 0 & s_y & y_f(1-s_y) \\ 0 & 0 & 1 \end{bmatrix}$$

$$= T(x_f, y_f) \cdot S(s_x, s_y) \cdot T(-x_f, -y_f) =$$

$$= S(x_f, y_f, s_x, s_y).$$

LectureNotes.in

# Reflection in Graphics

Reflection :-

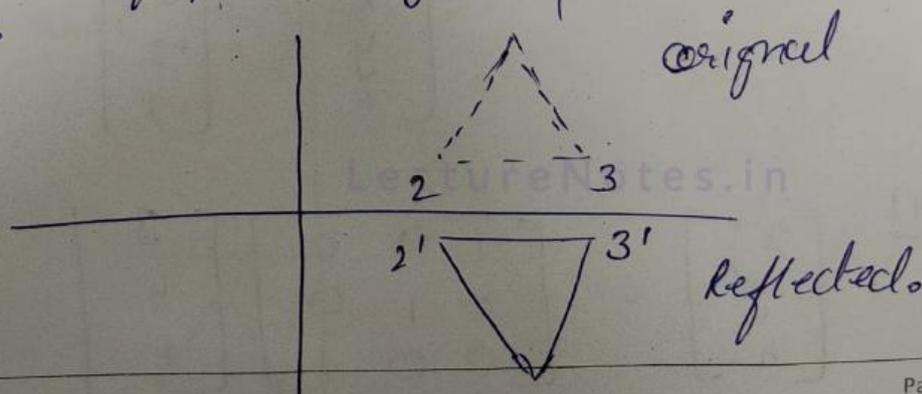
It is a transformation which produces mirror image of an object.

A mirror image is relative to axis of reflection by rotating the object about reflection axis  $180^\circ$ .

Reflection about the line  $y=0$ , the x-axis is with transformation matrix

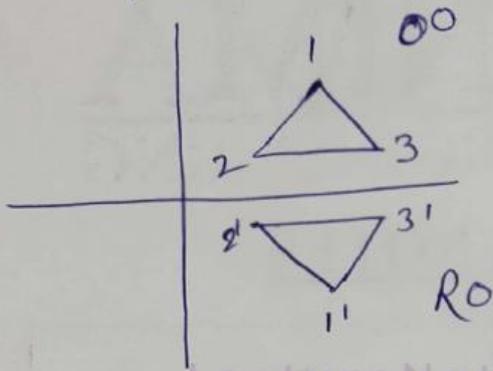
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This matrix flips the y value & keeping x as same.



Page No.:

① Reflection w.r.t x axis

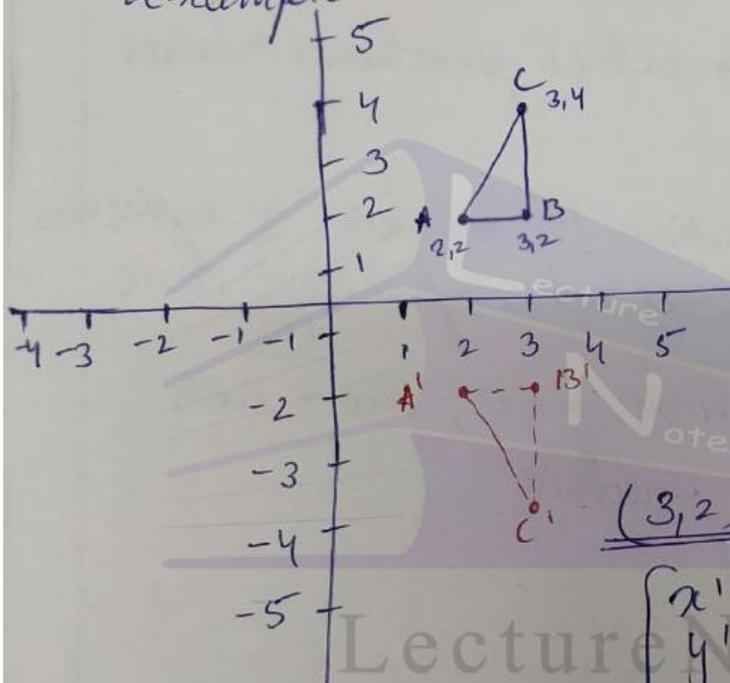


$$P' = R'P$$

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ h \end{bmatrix}$$

LectureNotes.in

Example 5-



(2,2)

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

(3,2)

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

(3,4)

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$$

Scanned by CamScanner

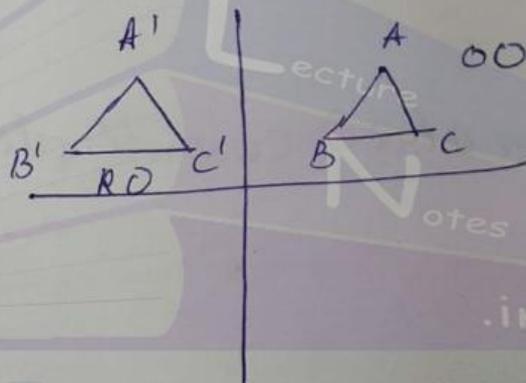
# Reflection in Graphics

New - Co-ordinates :-

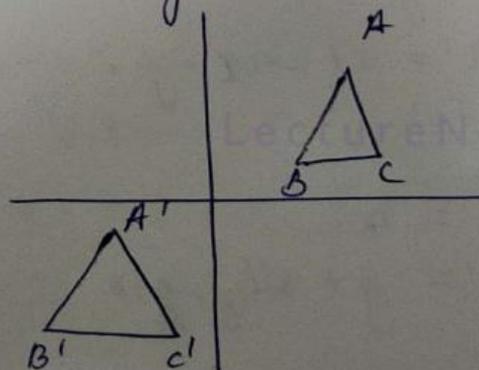
$$A'(2, -2) \quad B'(3, -2) \quad C'(3, -4)$$

Reflection w.r.t y axis :-

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ h \end{bmatrix}$$

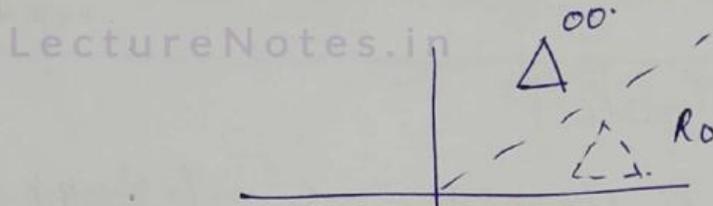


Reflection w.r.t origin :-



$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ h \end{bmatrix}$$

Reflection w.r.t, when  $y=x$ .



$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ h \end{bmatrix}$$

SHEARING :-

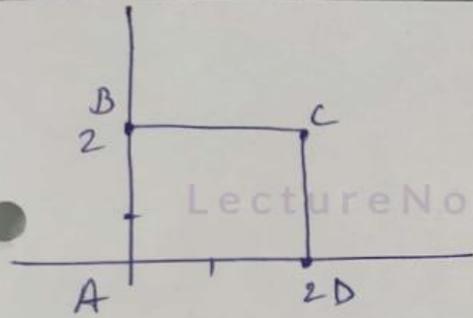
Same distort on other coordinate :-

x-shear } skewing.  
y-shear }

In x shear;  $y' = y$   
 $x' = x + k_x \cdot y$

In y shear;  $x' = x$   
 $y' = y + k_y \cdot x$

# Shearing in Graphics



$(0,0), (0,2), (2,2), (2,0)$

$S_h x = 2 \text{ units}$

LectureNotes.in

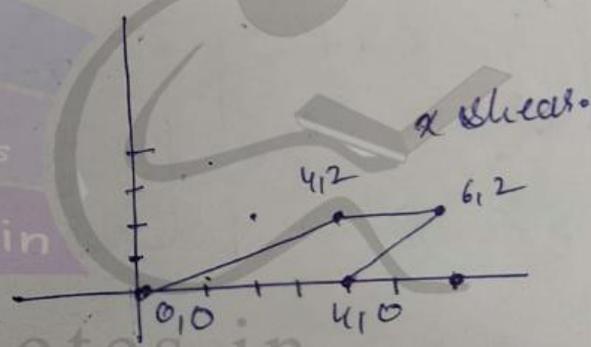
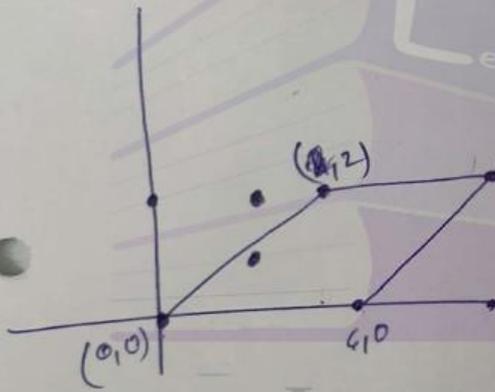
$$\begin{aligned} \underline{0,0} \\ = 0 + 2 \cdot 0 \\ = 0 \end{aligned}$$

$$\begin{aligned} \underline{0,2} \\ = 0 + 2 \cdot 2 \\ = 4 \end{aligned}$$

$$\begin{aligned} \underline{2,2} \\ = 2 + 2 \cdot 2 \\ = 6 \end{aligned}$$

$$\begin{aligned} \underline{2,0} \\ = 2 + 2 \cdot 0 \\ = 2 \end{aligned}$$

$(x', y') = (0,0), (4,2), (6,2), (2,0)$

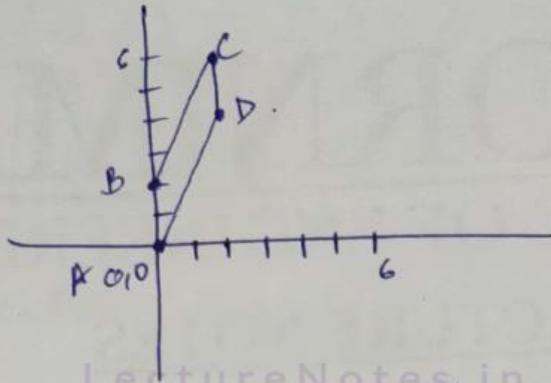


LectureNotes.in

y-shear

$S_h y = 2 \text{ units}$

A (0,0)	B (0,2)	C (2,2)	D (2,0)
$y + S_h y \cdot x$	$2 + 2 \cdot 0$	$2 + 2 \cdot 2$	$0 + 2 \cdot 2$
$0 + 2 \cdot 0$	2	6	4
$= 0$			
(0,0)	(0,2)	(2,6)	(2,4)



## MATRIX Representation :-

x shear :-

$$\begin{bmatrix} 1 & shx & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ h \end{bmatrix}$$

y shear :-

$$\begin{bmatrix} 1 & 0 & 0 \\ shy & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ h \end{bmatrix}$$



# *Computer Graphics And Multimedia*

Topic:  
*Polygon Filling Algorithm*

Contributed By:  
*Ankita Jiyani*

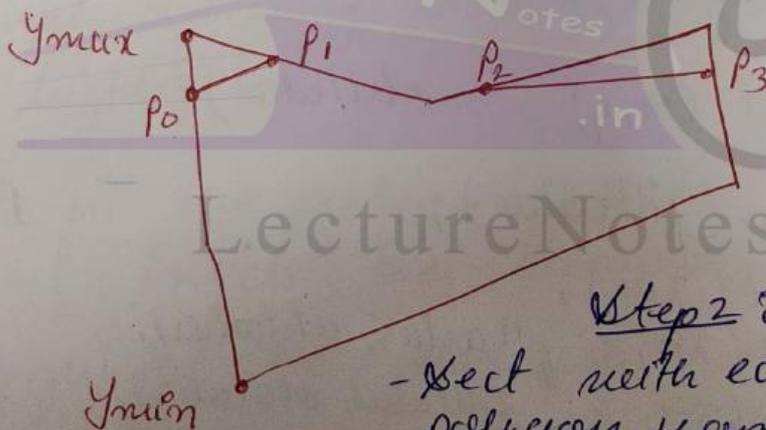
# Polygon Filling Algorithms

Polygon Filling Algo<sup>s</sup>-

To fill polygon with particular colors, you need to determine the pixels falling on the border of the polygon & those which fall inside the polygon.

Scan line Algorithm<sup>s</sup>-

This algo works by intersecting scan line with polygon edges & fills the polygon b/w pairs of intersection.



Step 1- find out  $Y_{max}$  &  $Y_{min}$  from the given below;

Step 2: Scan line intersect with each edge of the polygon from  $Y_{min}$  to  $Y_{max}$ . Name each intersection point of polygon -  $P_0, P_1, P_2, P_3$ .

Page No.:

Step 3: sort the intersection points in the increasing order of x-coordinates.  
( $p_0, p_1$ ) ( $p_1, p_2$ ) ( $p_2, p_3$ ).

Step 4: fill all these pairs of coordinates that are inside the polygon & ignore the alternate pairs.

Flood Fill Algorithm:-

Disadvantages of scan conversion, scan fill algorithms are:-

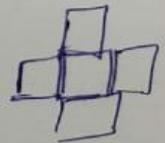
↳ More complex

↳ It is discrete algorithm

Sometimes we want to fill a color area that is not defined within a single color boundary

Procedure flood fill ( $x, y, \text{oldcolor}, \text{fillcolor}$ )

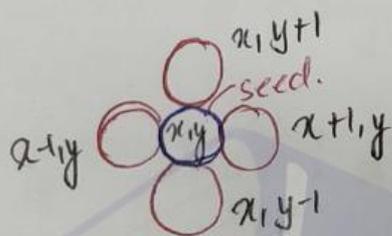
```
if (getpixel ( $x, y$ ) == oldcolor)
{
  setcolor (fillcolor);
  setpixel ( $x, y$ );
  floodfill4 ( $x+1, y, \text{fillcolor}, \text{oldcolor}$ );
  floodfill4 ( $x-1, y, \text{fillcolor}, \text{oldcolor}$ );
  floodfill4 ( $x, y+1, \text{fillcolor}, \text{oldcolor}$ );
  floodfill4 ( $x, y-1, \text{fillcolor}, \text{oldcolor}$ );
}
```



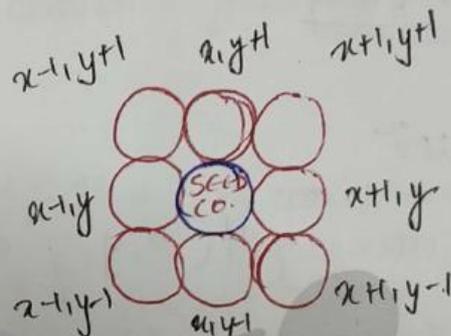
# Boundary Filling Algorithm

Example :-  
MS Paint.

## Boundary filling Algorithm.



4 way connected



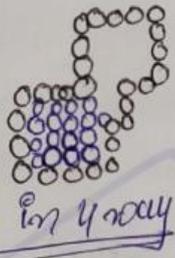
8 way connected.

- ↳ First boundary drawn, and boundary pixels are highlighted.
- ↳ To fill the polygon, select any one pixel inside the boundary.
- ↳ Highlight the connected pixel, until boundary pixel is reached.
- ↳ The procedure tests neighbouring pixels to determine whether they are of boundary color.

This can be done in 2 ways;  
4 way connected, 8 way connected.

An 8 way connected would correctly fill the interior of the area. A 4 way will fill it partially.

LectureNotes.in



Procedure:-

```
boundary-fill(x, y, f-color, b-color)
{
  if (getpixel(x, y) != b-color && getpixel(x, y) != f-color)
  {
    putpixel(x, y, f-color)
    boundary-fill(x+1, y, f-color, b-color)
    boundary-fill(x, y+1, f-color, b-color)
    boundary-fill(x-1, y, f-color, b-color)
    boundary-fill(x, y-1, f-color, b-color)
  }
}
```

# Difference between Seed Fill and Boundary Fill

## Disadvantages of seed fill.

- ↳ If an pixel is in some other color, then the fill terminates & polygon remain unfilled
- ↳ Seed fill method doesn't work for large polygons.

## Difference b/w Flood Fill & Boundary Fill.

### Flood Fill

colors the entire area in an enclosed figure through interconnected pixels using a single color.

So, Flood fill is one in which all connected pixel of a selected color get replaced by a fill color.

It can use unpredictable amount of memory to finish because it isn't known how many subfills will be spawned.  
Time consuming.

### Boundary Fill.

Area gets covered with pixels of a chosen color as boundary but's giving technique

when the color of boundary is found. It stops

Usually more complicated but it is a linear algo.

less time consuming.



# *Computer Graphics And Multimedia*

Topic:

*Window To Viewport Transformation*

Contributed By:

*Ankita Jiyani*

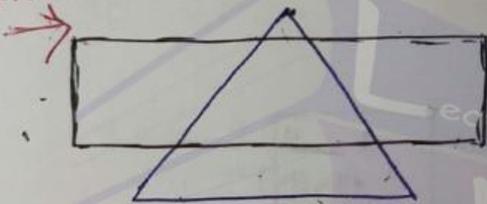
# Window to Viewport Transformations

Point & line Clipping :-

Clipping :- The objects to be displayed, which is inside the window.

First we will study : window to viewport transformation :-

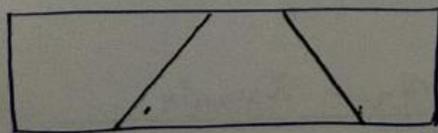
Window



→ we want to capture this image. or a portion of the image

→ The portion or part in which object or image is displayed, is a window.

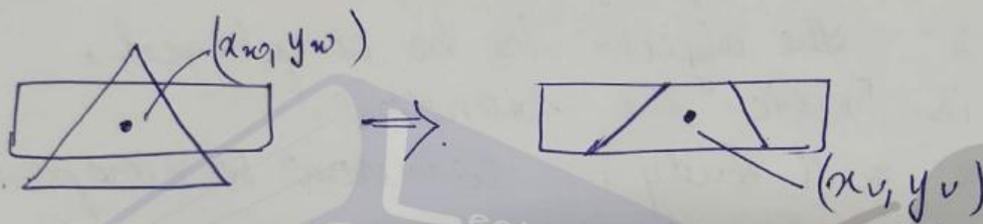
→ WINDOW - WHAT IS TO BE DISPLAYED.  
VIEWPORT - where object is displayed.



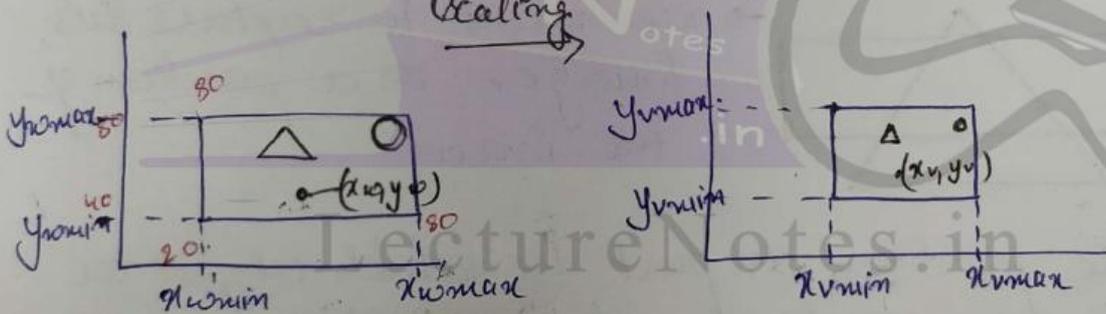
→ viewport

For a window, we have boundary values,  
 $x_{wmin}$ ,  $x_{wmax}$  → for x-axis  
 $y_{wmin}$ ,  $y_{wmax}$  → for y-axis. } window coordinates

For a viewport;  
 $x_{vmin}$ ,  $x_{vmax}$  → for x-axis. } device  
 $y_{vmin}$ ,  $y_{vmax}$  → for y-axis. } co-ordinates



window to viewport transformation;  
 Scaling



WINDOW

VIEWPORT.

The relative position is not changed.  
 Only size is changed.

$$x \text{ coordinate } \left\{ \frac{x_w - x_{wmin}}{x_{wmax} - x_{wmin}} = \frac{x_v - x_{vmin}}{x_{vmax} - x_{vmin}} \right.$$

$$y \text{ coordinate } \left\{ \frac{y_w - y_{wmin}}{y_{wmax} - y_{wmin}} = \frac{y_v - y_{vmin}}{y_{vmax} - y_{vmin}} \right.$$

# Window to Viewport Transformation Example

Example:-

$$x_{wmin} = 20$$

$$x_{wmax} = 80$$

$$y_{wmin} = 40$$

$$y_{wmax} = 80$$

$$(x_w, y_w) = (30, 80)$$

$$x_{vmin} = 30$$

$$x_{vmax} = 60$$

$$y_{vmin} = 40$$

$$y_{vmax} = 60$$

$$(x_v, y_v) = ?$$

$$\frac{30 - 20}{80 - 20} = \frac{x_v - 30}{60 - 30} \Rightarrow \frac{10}{60} = \frac{x_v - 30}{30}$$

$$x_v = 35$$

$$\frac{80 - 40}{80 - 40} = \frac{y_v - 40}{60 - 40} \Rightarrow \frac{40}{40} = \frac{y_v - 40}{20}$$

$$y_v = 20 + 40 = 60$$

$$(x_v, y_v) = (35, 60)$$

**Point Clipping:** Discarding the object which are outside of the window.

- ↳ clip a point.
- ↳ points lies outside the window are discarded.
- ↳ a point say  $(x, y)$  lies inside the window if it satisfies following property:-

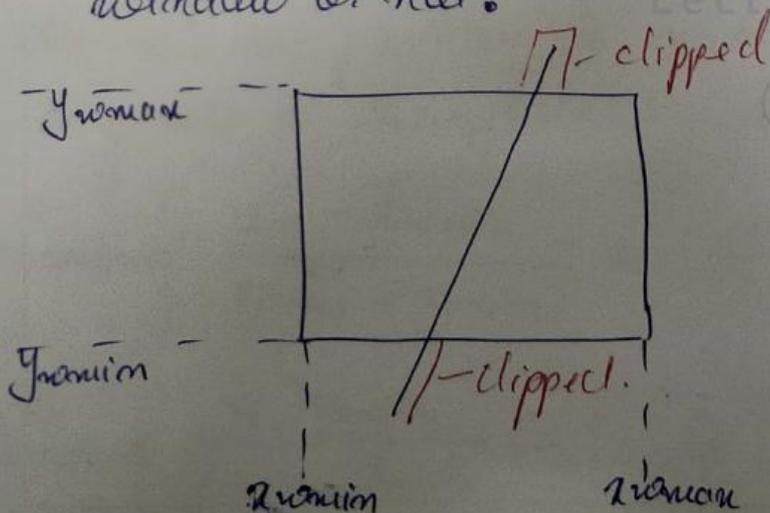
①  $x_{\min} \leq x \leq x_{\max}$

②  $y_{\min} \leq y \leq y_{\max}$

If it doesn't satisfy the above property, discard the point.

**Line Clipping:** Discarding the part of line which is outside the window.

There are various algorithm to check whether the portion of a line exist inside the window or not.



# Types of Clipping Algorithms

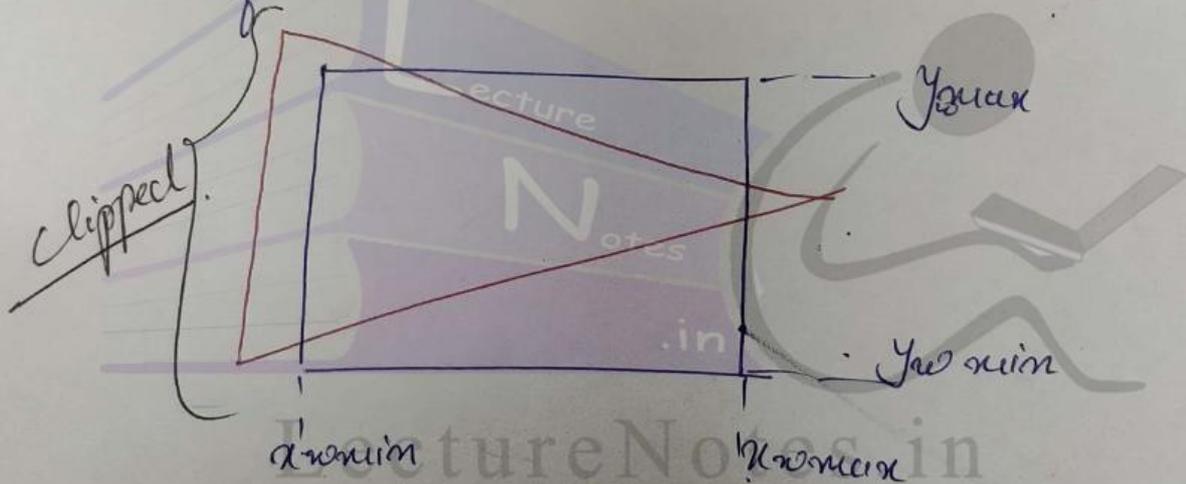
Algorithms are :-

- ① Cohen-Sutherland line clipping Algo
- ② Wang-Besky line clipping Algo.

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Polygon Clipping :-

Discarding the part of the polygon, outside of the window.



Algorithms are :-

- ① Sutherland-Hodgeman.



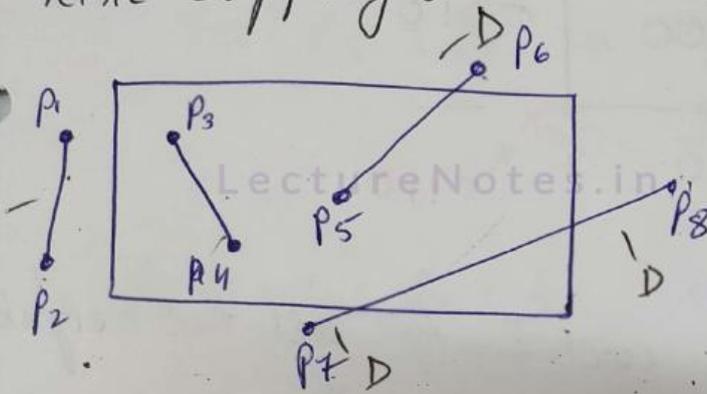
# *Computer Graphics And Multimedia*

Topic:  
*Line Clipping Algorithm*

Contributed By:  
*Ankita Jiyani*

# Line Clipping Algorithm

Line Clipping :-



Discard lines outside of the window.

- $P_1P_2$  - Reject  $\rightarrow$  check endpoints for every line.
  - $P_3P_4$  - consider
  - $P_5P_6$  - clipping reqd. if both points lie outside
  - $P_7P_8$  - clipping reqd. Reject the line.
- if both points inside  
accept the line.

But this method has disadvantage;

if  $P_7P_8 \rightarrow$  lies outside  $\rightarrow$  Reject.

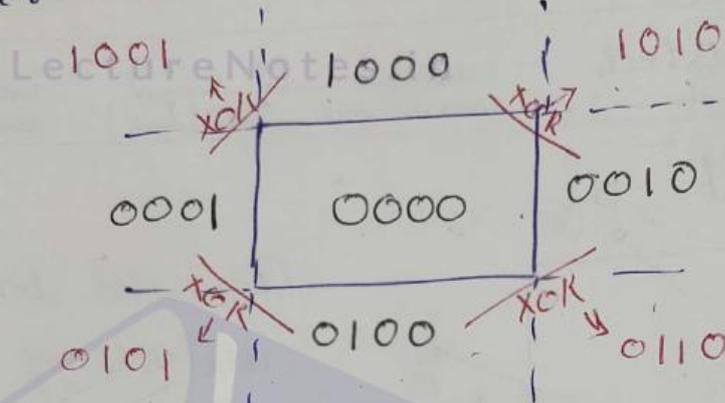
$P_5P_6 \rightarrow$  one point inside & one is outside.

is not efficient.

Page No.:

# COHEN-SUTHERLAND LINE CLIPPING ALGORITHM.

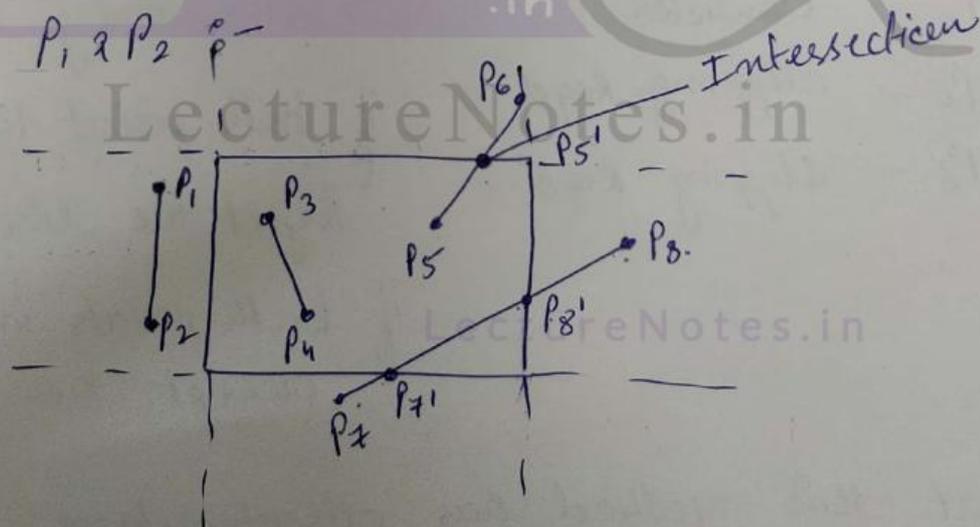
-> This algorithm is based on 4 bit region code.



Below  
A B R L - left  
Above Right

-> we have given the codes to all the neighbors of the window.

-> For  $P_1$  &  $P_2$



code for  $P_1$  &  $P_2$  -  $\begin{matrix} 0001 & - & P_1 \\ 0001 & - & P_2 \\ \hline 0001 & \text{non-zero.} \end{matrix}$

If result is non-zero, then line doesn't

# Cohen Sutherland Line Clipping Algorithm

lie in the region.

For  $P_3$  &  $P_4$  →

$$\begin{array}{r} 0000 \\ 0000 \text{ AND} \\ \hline 0000 \rightarrow \text{zero result.} \end{array}$$

$P_3, P_4$  lies in the region 0000.

For  $P_5$   $P_6$  →

$$\begin{array}{r} 0000 \text{ — zero.} \\ 1000 \text{ AND} \\ \hline 0000 \rightarrow \text{clipping required.} \end{array}$$

Some part lies inside & some outside.

→ Find intersection point on window.

$P_5'$   $P_6$  →

$$\begin{array}{r} 0000 \\ 0000 \\ \hline 0000 \rightarrow \text{All zero.} \end{array}$$

→  $P_5' - P_6$  → clipped.

~~Yes~~  $P_7 P_8$ .

$$\begin{array}{r} 0100 \\ 0010 \\ \hline 0000 \end{array} \rightarrow \text{zero Result.}$$

$$P_7' P_8' = \begin{array}{r} 0000 \\ 0000 \\ \hline 0000 \end{array} \rightarrow \text{zero Result.}$$

$$P_7 P_8' = \begin{array}{r} 0100 \\ 0000 \\ \hline 0000 \end{array} \rightarrow \text{zero Result.}$$

Pairs.	Region code	Result.	Intersection point	Decision.
$P_1 P_2$	0001 $\wedge$ 0001	0001	No.	Reject.
$P_3 P_4$	0000 $\wedge$ 0000	0000	No	Accept.
$P_5 P_6$	0000 $\wedge$ 1000	0000	$P_5'$	$P_5' P_6$ Clipped.
$P_7 P_8$	0100 $\wedge$ 0010	0000	$P_7'$	$P_7 P_7'$ Clipped.
$P_7' P_8$	0000 $\wedge$ 0010	0000	$P_8'$	$P_8' P_8$ Clipped.

# Pseudo code for Cohen Sutherland Algorithm

Pseudo code :-

- Step 1 - Assign a region code for 2 endpoints.
- Step 2 - If both endpoints have region code, 0000 then given line is completely inside.
- Step 3 - else perform logical AND operation for both endpoints.
- Step 3.1 : if result is non zero, then line is completely outside.
- Step 3.2 : else line is partially inside.
- 3.2.1 : choose end point that is outside the given rectangle.
- 3.2.2 : find intersection point of the rectangular boundary.
- 3.2.3 : replace end point with intersection point & update the region code.
- 3.2.4 : Repeat step 2 until we find a clipped line.
- Step 4 - Repeat Step 1 for other lines.

Questions :-

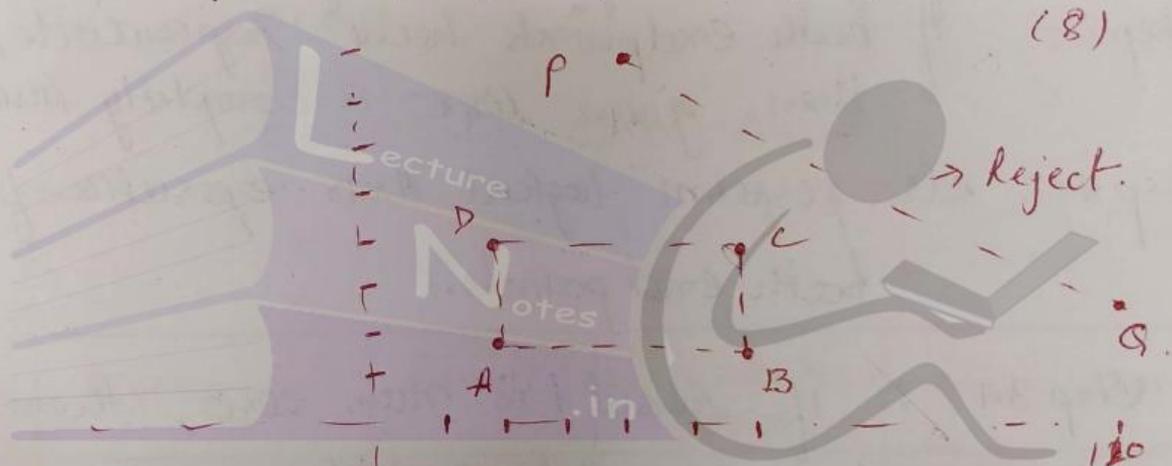
Q1) use Cohen-Sutherland line clipping algo to find the visible portion of the line  $P(40, 80), Q(120, 30)$  inside the window.

the window is defined as  $A(20, 20)$

$B(60, 20), C(60, 40)$  &  $D(20, 40)$ . RTO-2013.

(8)

Sol

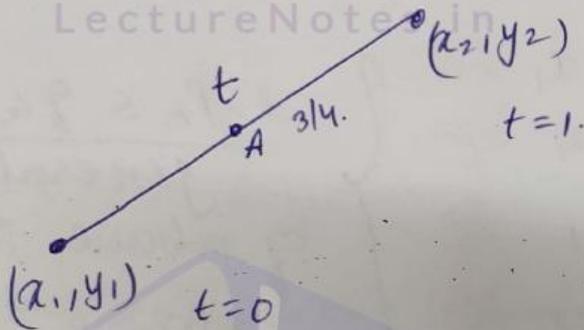


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# Liang Basky Line Clipping Algorithm

LIANG-BASKY LINE CLIPPING ALGORITHM.



line A travel from  $(x_1, y_1)$  to  $(x_2, y_2)$  in time interval  $t=0$  to  $t=1$ .

$$x = t_0 x_2 + (1-t) x_1$$

$$\frac{1}{4} x_1 + \frac{3}{4} x_2$$

$$= x_1 + t(x_2 - x_1)$$

$$\boxed{x = x_1 + t(\Delta x)}$$

find x coordinate at particular time interval  $t$ ,  $0 < t < 1$

$$\boxed{y = y_1 + t * \Delta y}$$

$$0 < t < 1$$

The intermediate point is the intersection point on the window.

The linear search algorithm also considers following properties:-

$$x_{\min} \leq x_1 + t \Delta x \leq x_{\max}$$

$$y_{\min} \leq y_1 + t \Delta y \leq y_{\max}$$

$$t \Delta x \geq x_{\min} - x_1 \quad \text{--- (1)}$$

$$t \Delta x \leq x_{\max} - x_1 \quad \text{--- (2)}$$

$$t \Delta y \geq y_{\min} - y_1 \quad \text{--- (3)}$$

$$t \Delta y \leq y_{\max} - y_1 \quad \text{--- (4)}$$

of the form  $t \Delta x \leq q_k$   
 general form  
 of above inequalities  
 $k=1, 2, 3, 4.$

eq 1 & eq 3 should be multiplied with (-1)

$$- t \Delta x \leq x_1 - x_{\min}$$

$$t \Delta x \leq x_{\max} - x_1$$

$$- t \Delta y \leq y_1 - y_{\min}$$

$$t \Delta y \leq y_{\max} - y_1$$

All inequalities are converted into lesser than equal to.

$$P_1 = -\Delta x$$

$$P_2 = \Delta x$$

$$P_3 = -\Delta y$$

$$P_4 = \Delta y.$$

$$Q_1 = x_1 - x_{\min}$$

$$Q_2 = x_{\max} - x_1$$

$$Q_3 = y_1 - y_{\min}$$

$$Q_4 = y_{\max} - y_1$$

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If  $P_k = 0 \rightarrow$  line is parallel to window

Corresponding  $q_k < 0$  - line is outside

$q_k > 0$  - line is inside. or,  
partially inside.

$q_k = 0$  - within the boundary. or,  
partially inside.

If  $P_k < 0 \rightarrow$  find  $(x, y)$  at  $t$ .

$$t_1 = \max\left(0, \frac{q_k}{P_k}\right) \quad \text{// Intersection point.}$$

$$x = x_1 + t_1 \Delta x$$

$$y = y_1 + t_1 \Delta y.$$

If  $P_k > 0$ , time interval at  $t_2 \parallel 1$

$$t_2 = \min\left(1, \frac{q_k}{P_k}\right) \parallel \text{intersection point.}$$

$$x = x_1 + t_2 \cdot \Delta x$$

$$y = y_1 + t_2 \cdot \Delta y$$

If  $t_1 = 0 \rightarrow$  doesn't change

no change in initial point.

A initial point is inside the window

If  $t_1 \neq 0 \rightarrow$  changed.

Change in initial point.

A final or end point doesn't lie in the window.

If  $t_2 = 1$ , second point inside the window.

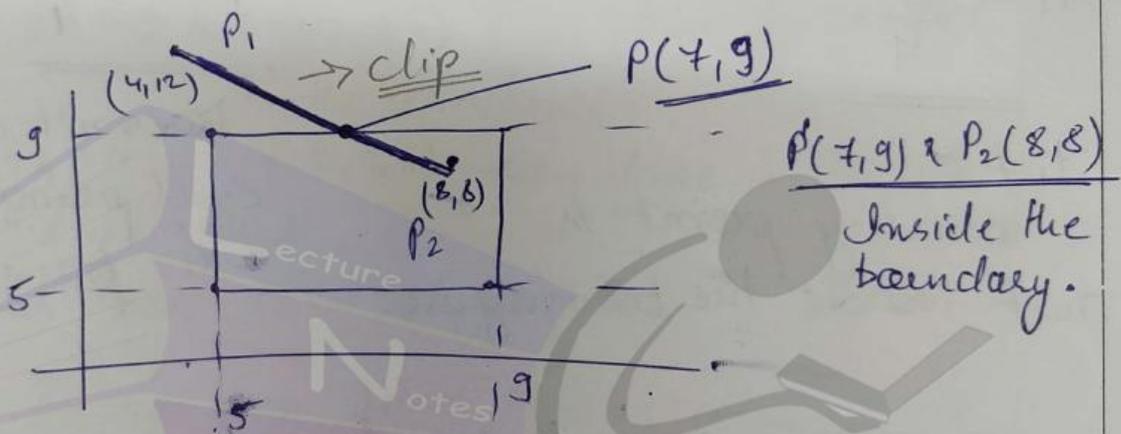
If  $t_2 \neq 1$ , if not outside the window.

# Liang Basky Example

Example :-

$x_{\min} = 5$        $y_{\min} = 5$   
 $x_{\max} = 9$        $y_{\max} = 9$

Line points  $P_1(4, 12)$        $P_2(8, 8)$



$$P_1 = -4$$

$$Q_1 = 4 - 5 = -1$$

$$P_2 = 4$$

$$Q_2 = 9 - 4 = 5$$

$$P_3 = -(4) = 4$$

$$Q_3 = 12 - 5 = 7$$

$$P_4 = -4$$

$$Q_4 = 9 - 12 = -3$$

For  $P_1$  &  $P_4$ ,

$$P_k < 0;$$

$$t_1 = \max\left(0, \frac{Q_1}{P_1}, \frac{Q_4}{P_4}\right)$$

$$\underline{P_2 \text{ \& } P_3 > 0}$$

$$t_2 = \min\left(1, \frac{Q_2}{P_2}, \frac{Q_3}{P_3}\right)$$

$P_1 \text{ \& } P_4$

$$t_1 = \max\left(0, \frac{1}{4}, \frac{3}{4}\right)$$

$$t_1 = \max\left(0, \frac{1}{4}, \frac{3}{4}\right)$$

$$\underline{\underline{t_1 = \frac{3}{4}}}$$

$$t_1 \neq 0$$

starting point is  
not inside the boundary.

$P_2 \text{ \& } P_3$

$$t_2 = \min\left(1, \frac{5}{4}, \frac{7}{4}\right)$$

$$\underline{\underline{t_2 = 1}}$$

Doesn't change  
end point inside  
the boundary.

$$x = x_1 + t_1 \cdot \Delta x$$

$$4 + \frac{3}{4} \times 4$$

$$= 7$$

$$y = y_1 + t_1 \cdot \Delta y$$

$$12 + \frac{3}{4} \times 4 = 12 - 3$$

$$= 9$$

$$(x, y) = (7, 9)$$

Values at  $t_1 = \frac{3}{4}$ ,  $(x, y) = (7, 9)$

# Pseudo Code for Liang Basky

Pseudo Code :-

Step 1 :- Get the line endpoints  $(x_1, y_1)$  to  $(x_2, y_2)$

Step 2 :- Find  $Ax, Ay, P_1, \dots, P_4, Q_1, \dots, Q_4$

Step 3 Assign  $t_1 = 0$ , &  $t_2 = 1$

- i) if  $P_k = 0$  ( $k=1, 2, 3, 4$ ) then line is parallel to the window.
- ii) if  $q_k < 0$  ( $k=1, 2, 3, 4$ ) then line is outside the window.

iii) For non-zero value of  $P_k$ .

if  $P_k < 0$  then find  $t_1$   
 $t_1 = \max(0, q_k/P_k)$

else  $P_k > 0$  then find  $t_2$

$t_2 = \min(1, q_k/P_k)$

if  $t_1 > t_2$  then line is completely outside - reject.

or.

else find new set of  $(x, y)$  if  $t_1, t_2$  is changed.

$$x = x_1 + t \cdot \Delta x$$

$$y = y_1 + t \cdot \Delta y$$

Solution  $P_1(-1, -2)$

$P_2(2, 4)$

$$2 - (-1)$$

$$2 + 1 = 3$$

$$4 - (-2) = 4 + 2 = 6$$

$$x_{\text{min}} = 0 \quad y_{\text{min}} = 0$$

$$x_{\text{max}} = 1 \quad y_{\text{max}} = 1$$

$$P_1 = -3$$

$$P_2 = 3$$

$$P_3 = -6$$

$$P_4 = 6$$

$$Q_1 = -1 - 0 = -1$$

$$Q_2 = 1 - (-1) = 2$$

$$Q_3 = -2 - 0 = -2$$

$$Q_4 = 1 - (-2) = 3$$

$$Q_1/P_1 = 3/3$$

$$Q_2/P_2 = 2/3$$

$$Q_3/P_3 = 1/3$$

$$Q_4/P_4 = 1/2$$

$$P_1, P_3 < 0$$

$$P_2, P_4 > 0$$

$$t_1 = \max\left(0, \frac{1}{3}, \frac{1}{3}\right)$$

$$t_2 = \min\left(1, \frac{2}{3}, \frac{1}{2}\right)$$

$$t_1 = \frac{1}{3}$$

$$t_2 = \frac{1}{2}$$

$$t_1 \neq 0$$

$$t_2 \neq 0$$

$$x = -1 + \frac{1}{3} \cdot 3 = 0$$

$$x = -1 + \frac{1}{2} \cdot 3 = \frac{1}{2}$$

$$y = -2 + 6 \cdot \frac{1}{3} = 0$$

$$y = -2 + \frac{1}{2} \cdot 6 = 1$$

$$(x, y) = (0, 0)$$

$$(x, y) = \left(\frac{1}{2}, 1\right)$$

Point inside the region is

$$P_1(0, 0)$$

$$P_2\left(\frac{1}{2}, 1\right)$$



# *Computer Graphics And Multimedia*

Topic:

*Polygon Clipping Algorithm*

Contributed By:

*Ankita Jiyani*

# Sutherland Hodgeman Polygon Clipping Algorithm

## SUTHERLAND - HODGEMAN POLYGON CLIPPING ALGORITHM.

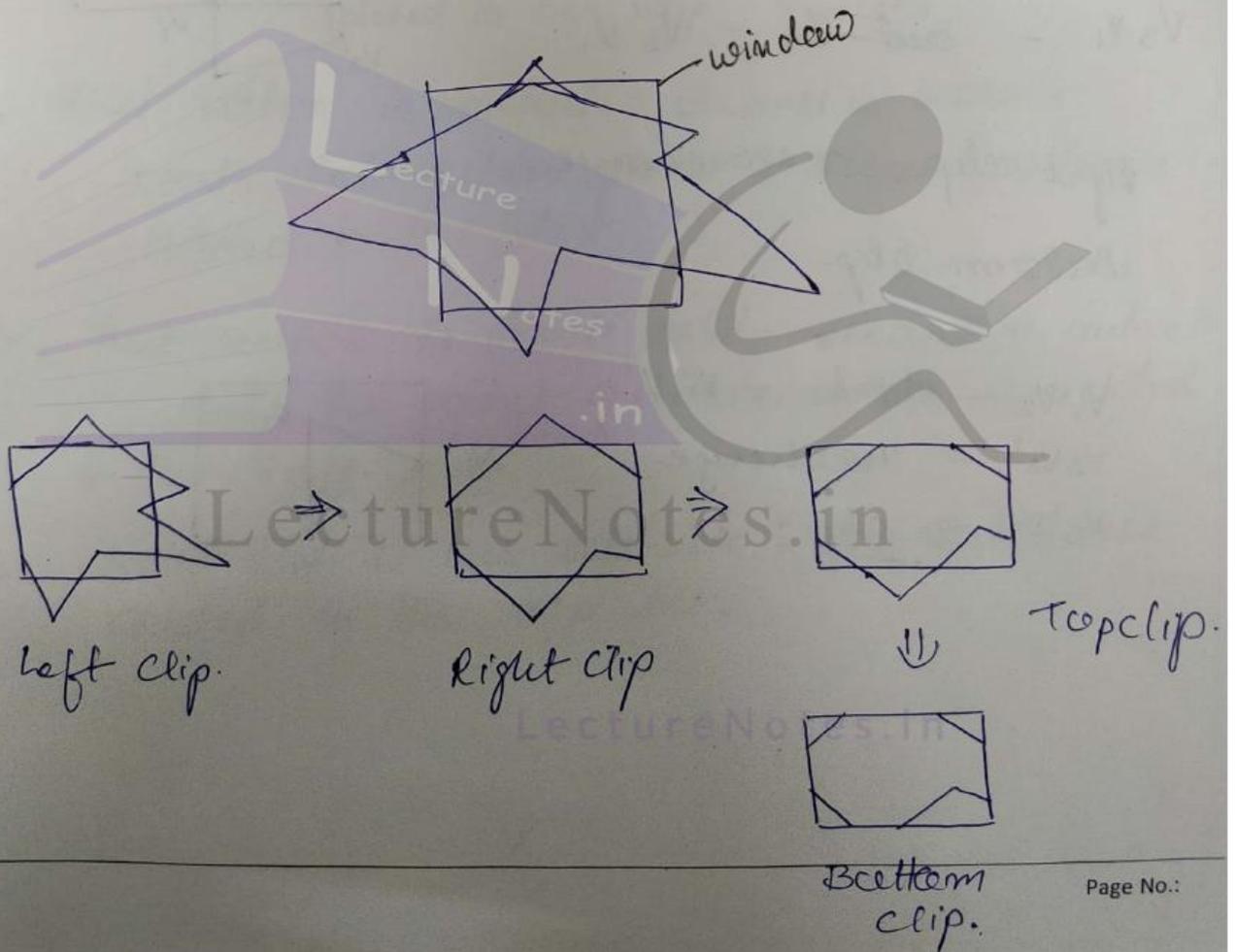
Four procedures;

↳ left clip.

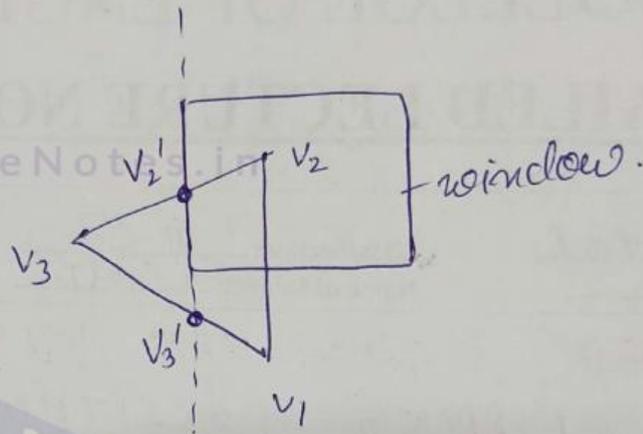
↳ right clip.

↳ top clip.

↳ Bottom clip.



one output is input to the other clip function.

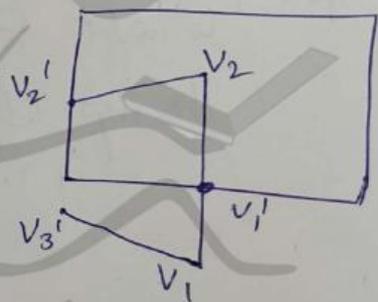


left clip

$V_1 V_2$  - In - In -  $V_2$

$V_2 V_3$  - In - out -  $V_2'$

$V_3 V_1$  - out - In -  $V_3' V_1$



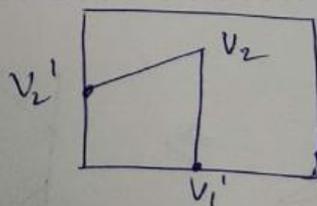
right-clip.  $\rightarrow$  no change.

Bottom-clip.

$V_1 V_2$  - out - In -  $V_1 V_1'$

$V_2 V_2'$  - no change

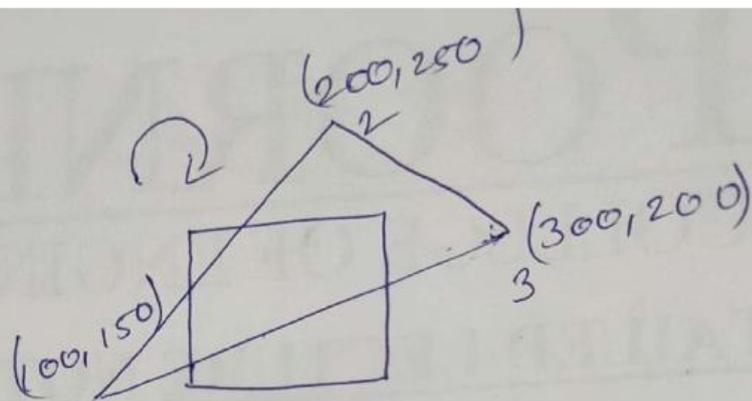
$V_3' V_1 \rightarrow$  clip



Clip polygon using Sutherland-Hodgeman Algo,  
 Input is in the form of vertices of the polygon  
 in clockwise order.

There are 4 possible cases for any given edge of  
 given polygon against current clipping edge  $e$ .

1. Both vertices are inside  $\therefore$  only second vertex  
 added to the o/p list.
2. First vertex is outside & second is inside  $\therefore$   
 both intersection point & second vertex is  
 added to the list.
3. First vertex is inside while second is outside;  
 only the point of intersection is added  
 to the o/p list.
4. Both vertices are outside  $\therefore$  no vertex are  
 added to the o/p list.



vertex list  $(100, 150)$   $(200, 250)$   $(300, 200)$

a polygon start from  $(x_1, y_1)$  & ends at  $(x_2, y_2)$ . This for clipper polygon.

$$P = (x_2 - x_1)(y - y_1) - (y_2 - y_1)(x - x_1)$$

if  $P < 0$ , point is right side of the line

$P = 0$ , point is on the line

$P > 0$ , point is left side of the line.



# *Computer Graphics And Multimedia*

Topic:  
*Hidden Surface Problem*

Contributed By:  
*Ankita Jiyan*

# Hidden surface problem

## Introduction:

When we view a picture containing non-transparent objects & surfaces, then we can't see those objects from view which are behind from objects closer to eye. We must remove these hidden surfaces to get a realistic screen image. The identification & removal of these surfaces is called Hidden surface problem.

The HSP is the process to determine which surfaces and parts of the surfaces are not visible from certain viewpoint.

There are 2 approaches for removing hidden surface problems:

↳ object space method.

This method is implemented in physical coordinate system.

An object-space method compares objects & part of objects to each other within the scene definition to determine which surfaces as a whole we should label as visible.

↳ Image space method.

It is implemented in screen coordinate systems.

In this method, visibility is decided point-by-point at each pixel position on the projection plane.

Most of the algorithms use image space method for detecting visible surface.

Algo. uses image space method, is much more efficient than object space method.

But object space methods are much more functional than image space methods.

# Image & Object Space Method

PAGE NO. ....

Image Space method :-

The representation of graphics in form of pixels is very flexible as raster systems keeps refreshing the screen by taking the values stored in frame buffer. Image space algorithms are very simple & efficient as their data structure is very similar to that of frame buffer. Commonly used algo is Z-Buffer Algo.

Object space method :-

Object space algo has advantage of retaining the relevant data & because of this ability the interaction of algo with the object becomes easier. The calculation done for the color is done only once. This algo also allows shadow generation to increase the depth of the 3D object on the screen. Easy to implement as  $\approx 20$  rather than  $4/20$

## Difference between Image and Object Space Method

### Object space model

This model defines the object with real world dimensions & volume.

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The dimensions may be in km, m or even in microns.

Less efficient than image space method.

More functional than image space method.

Color calculation is done only one time & is retained by it.

### Image space Model.

This model defines the object with dimensions of image, created to be displayed on the display device.

It makes use of scaled dimensions.

More efficient than object space method.

Less functional than object space method.

Color calculation once done & is overwritten later.

## Object Space Method Pseudo code

Object space method :-

For each object in the scene do;

Begin;

1. Determine those parts of object whose view is unobstructed by other parts of it or any other object with respect to viewing specification.
  2. Draw those parts in the object color.
- End.

- compare each object with all other objects to determine the visibility of object parts.
- If  $n$  objects in scene,  $O(n^2)$  - complexity.
- calculations performed at the resolution in which objects are defined.
- Computationally expensive than image space because step 1 is more complex.
- suitable for scene with smaller no. of objects.

## Image Space Method Pseudo code

Image space method :-

Begin;

1. Determine the object closest to the viewer that is pierced by the projector through the pixel

2. Draw the pixel in the object color.

End.

→ For each pixel, all  $n$  objects to determine the one closest to the viewer.

→ If there are  $p$  pixels in an image, then complexity is  $O(np)$

→ Accuracy is bounded by the display resolution.

→ A change in resolution requires re-calculation.



# *Computer Graphics And Multimedia*

Topic:

*Visible Surface Detection Algorithm*

Contributed By:

*Ankita Jiyani*

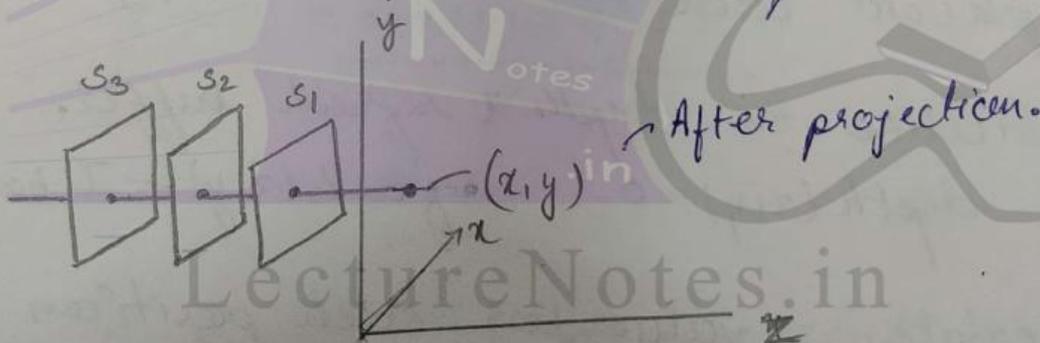
# Visible Surface Detection Algorithm

## Depth Buffer or Z Buffer Method

Visible surface detection Algorithm;

Depth Buffer Method or Z Buffer Method

- This is based on Image space method.
- It will compare surface depth at each pixel position on projection plane.
- Calculate the intensity value of the surface which is ~~is~~ nearer to view plane.

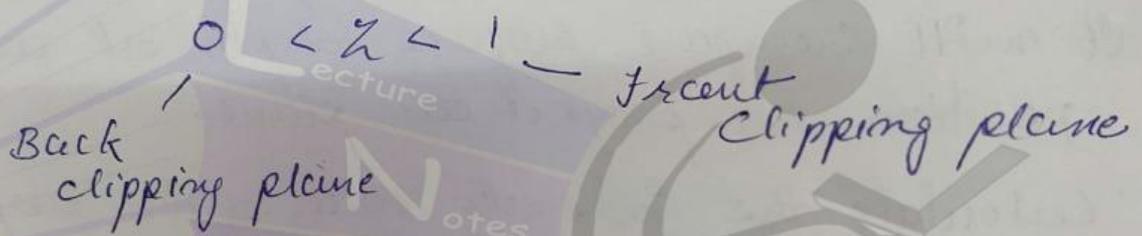


$S_1$  surface is nearer to the  $xy$  plane.

# Z Buffer Algorithm

Algorithm :-

- we are using 2 buffers,
- ↳ one buffer for storing  $z$ -value.
  - ↳ one buffer for refreshing, & storing intensity value of  $x, y$  coordinate for which we have calculated  $z$  value.
  - ↳ we are taking normalized value for  $z$ -coordinate.



Algo pseudo code :-

1. Initialize the depth & refresh buffer.  
 $depth(x, y) = 0$ .  $refresh(x, y) = I_{background}$
2. calculate  $z$  value for each position in the surface & then  
if  $z > depth(x, y)$   
set,  $depth(x, y) = z$ , &  
 $refresh(x, y) = I_{surf}(x, y)$

3. After processing all the surfaces, we will get visible surface in depth  $(x, y)$  and intensity values in  $z$  refresh  $(x, y)$ .

$z$  buffer or  $z$  value is calculated as;

$$Ax + By + Cz + D = 0$$

$$z = \frac{-Ax - By - D}{C}$$

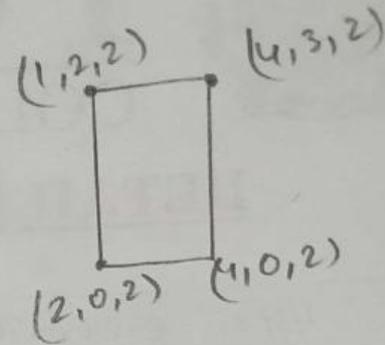
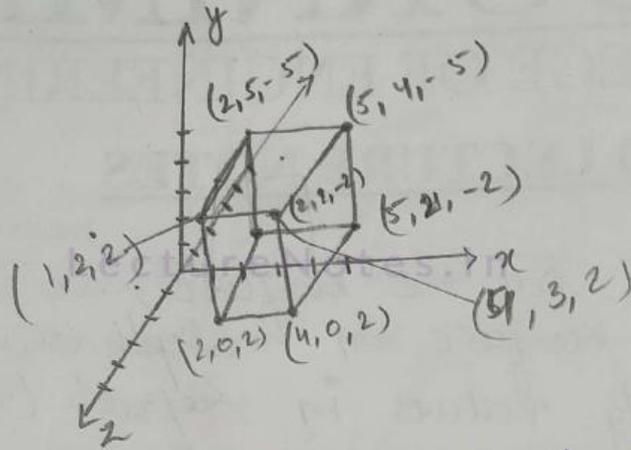
$$z' = \frac{-A(x+1) - By - D}{C}$$

$$z' = z - \frac{A}{C}$$

→ It can't be use for transparent objects.

# Z Buffer Algorithm Example

Example :-



$$\text{depth}(x, y) = 0.$$

Initialize  
depth(x, y)  
= 0.

0	0	0
0	0	0
0	0	0
0	0	0
0	0	0

$$z = 2.$$

$$z > 0.$$

update.  
depth(x, y) = 2.

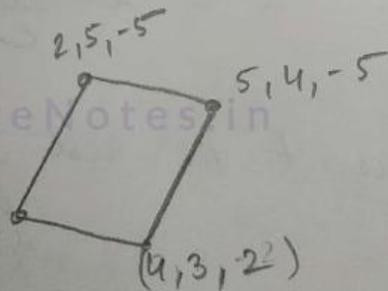
2	2	2
2	2	2
2	2	2
2	2	2
2	2	2

$$\text{depth}(x, y) = 2.$$

-5	-5	-5
-4	-4	-4
-3	-3	-3
-2	-2	-1
-1	-1	-1
0	0	0
1	1	1
2	2	2

depth buffer(x, y)

$$(1, 2, 2)$$





# *Computer Graphics And Multimedia*

Topic:  
*Back Face Detection*

Contributed By:  
*Ankita Jiyani*

# Back Face Detection Method

## BACK FACE DETECTION

It is method to detect visible surfaces.  
It is based on inside-outside test  
which is defined as:-

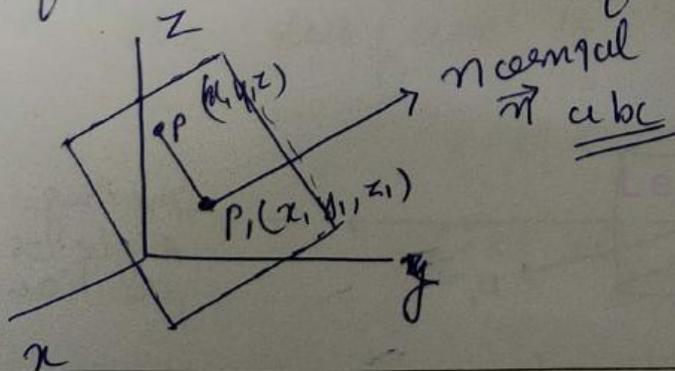
$$Ax + By + Cz + D < 0. \quad - (1)$$

The equation of normal plane is:-

$$Ax + By + Cz + D = 0. \quad - (2)$$

A point  $A(x, y, z)$  is inside the polygon  
if it satisfies equation 1.

If it doesn't satisfy, the point is outside.



$$\vec{P} - \vec{P}_1 = (x - x_1, y - y_1, z - z_1)$$

Since  $\vec{P} - \vec{P}_1$  &  $\vec{n}$  are perpendicular to each other.

Dot product is 0.

$$\vec{n} \cdot (\vec{P} - \vec{P}_1) = 0.$$

This dot product becomes the eq. of plane.

$$(a, b, c) \cdot (x - x_1, y - y_1, z - z_1) = 0$$

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

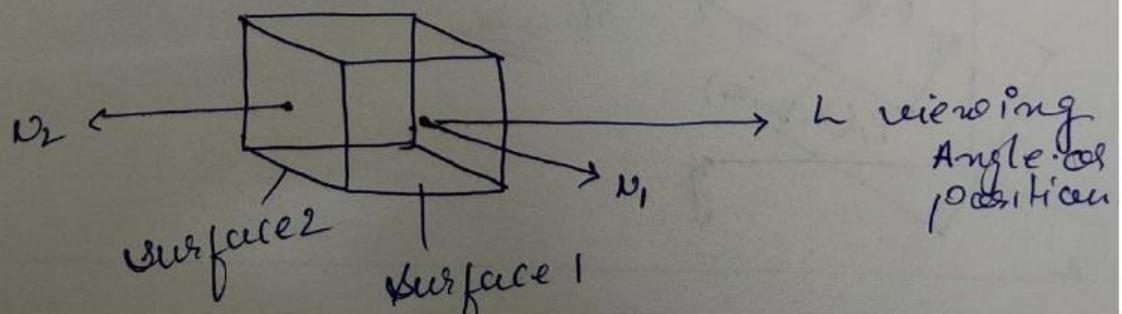
$$ax + by + cz - (ax_1 + by_1 + cz_1) = 0$$

$$ax + by + cz + d = 0.$$

$$d = -(ax_1 + by_1 + cz_1)$$

**BACK FACE PROCEDURE :-**

Consider the 2 faces (Surface) of a cube & their normal vectors



Now Depending on the angle between  $L$  and  $N_1$  &  $L$  and  $N_2$

The surface  $S_1$  &  $S_2$  may or may not be visible to the viewer.

→ The surface is visible from the position  $L$ ,  
iff.

$$0 \leq \theta \leq 90^\circ$$

or

$$0 \leq \cos \theta \leq 1$$

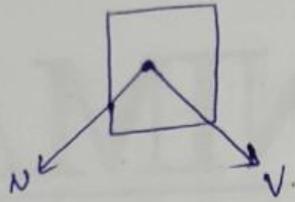
→ Calculate  $\cos \theta$  using

$$L \cdot N = |L| |N| \cos \theta.$$

→ The surface is visible; iff,

$L \cdot N < 0$ . Front face.

$L \cdot N > 0$  Back face.

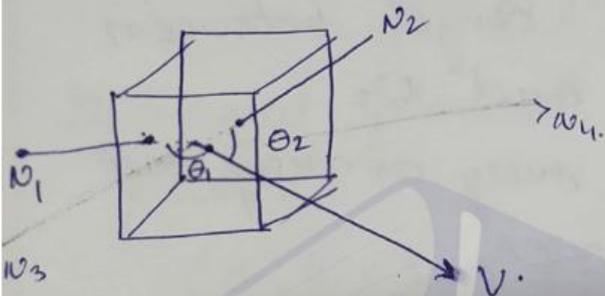


$$0 \leq \cos \theta \leq \pi/2$$

$$\cos \theta > 0.$$

It is Back face of the object.

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$$V \cdot N_2 = |V| |N_2| \cos \theta_2$$

$$V \cdot N_1 = |V| |N_1| \cos \theta_1$$

$$\text{So, } \cos \theta_1 \text{ \& } \cos \theta_2$$

are  $> 0$ .

So, these are Back faces

$$V \cdot N_3 = |V| |N_3| \cos \theta_3$$

$$V \cdot N_4 = |V| |N_4| \cos \theta_4.$$

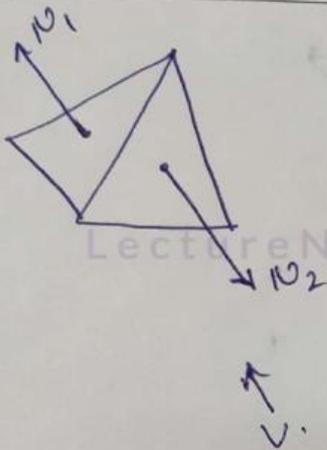
So, here  $\cos \theta_3 \text{ \& } \cos \theta_4 < 0$ .

So, these are the front faces.

$$\text{If } \pi/2 \leq \cos \theta < \pi$$

$$\cos \theta < 0$$

It will be considered as front face of the object.



$N_1$  &  $V$  are both in same direction.

So, here

$$V \cdot N_1 > 0.$$

Back face surface.

$N_2$  &  $V$  are in opposite direction

$$V \cdot N_2 < 0$$

Front face surface.

Now we are projecting the object into 2D and our viewing is along the  $z$  axis.

So, view coordinate will be,

$$V = (0, 0, V_z) \quad N = (A, B, C)$$

$V \cdot N$  will become -

$$V \cdot N = A \cdot 0 + B \cdot 0 + V_z \cdot C.$$

$$V \cdot N = V_z \cdot C \quad \left( \text{It depends on, sign of } C \right).$$



# *Computer Graphics And Multimedia*

Topic:

*Bezier Curve And It's Properties*

Contributed By:

*Ankita Jiyan*

# Bezier Curves & it's properties

Bezier Curves:-

• Properties:-

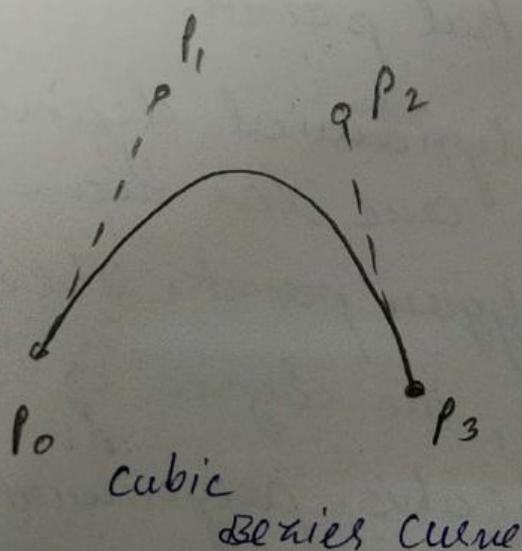
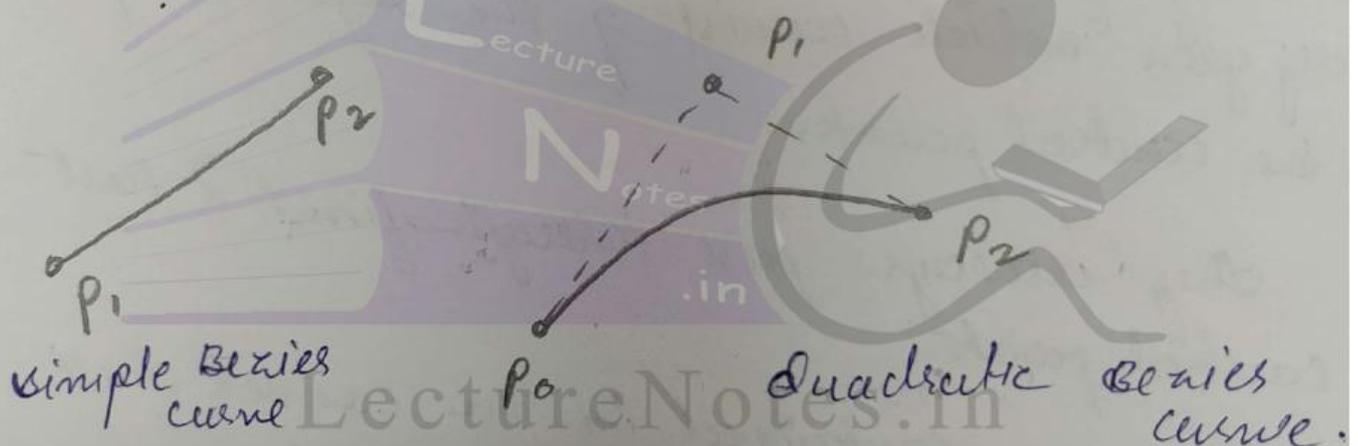
1. They follow the shape of the central polygon which consist of the segments joining the central points.
2. They always pass through first & last central points.
3. They are contained in the convex hull of their defining central points.
4. The degree of the polynomial defining the curve segment is one less than the no. of defining polygon point. Therefore for 4 central points, the degree of the polynomial is 3. i.e cubic polynomial.

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5. A Bezier curve generally follows the shape of defining polygon.

6. Bezier curves exhibit global control, means moving a control point alters the shape of a curve.

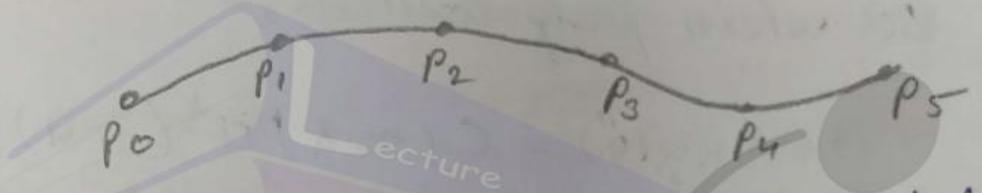
7. These curves are generated under the control of other points.



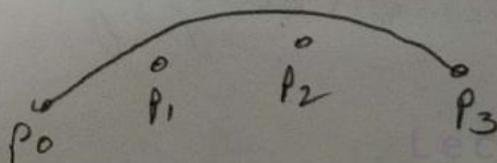
# Interpolation and Approximation Curve

There are two types of curves;

- 1) Interpolation curve; All points of the basic figure are located on the created figure i.e. interpolation curve segment.



- 2) Approximation curve; All points of the basic figure need not be located on the created figure i.e. approximation curve segment.



Bezier curve is the approximation curve.

## Blending Function for Bezier Curve

Functions of Bezier curve,  
Suppose we have  $n+1$  control points.

position is  $P_k = (x_k, y_k, z_k)$   
with  $k = 0$  to  $n$

These co-ordinates can be blended to produce the vector  $P(u)$  which describes the path of an approximation Bezier polynomial function between  $P_0$  &  $P_n$ .

$$P(u) = \sum_{k=0}^n P_k B_{k,n}(u), \quad 0 \leq u \leq 1$$

The  $B_{k,n}(u)$  is Bezier Blending function, and Bernstein polynomials:-

$$B_{k,n}(u) = C(n, k) u^k (1-u)^{n-k}$$

where,  $C(n, k)$  are the binomial coefficients.

$$C(n, k) = \frac{n!}{k!(n-k)!}$$

we can define Bezier Blending function with recursive calculation:-

$$B_{k,n}(u) = (1-u) B_{k,n-1}(u) + u B_{k-1,n-1}(u)$$

$$n > k \geq 1$$

## Example for Bezier Curve

with  $BEZ_{x,k} = u^k$  &  $BEZ_{y,k} = (1-u)^k$ ,  
the set of parametric equation,

$$x(u) = \sum_{k=0}^n x_k BEZ_{x,k,n}(u)$$

$$y(u) = \sum_{k=0}^n y_k BEZ_{y,k,n}(u)$$

$$z(u) = \sum_{k=0}^n z_k BEZ_{z,k,n}(u).$$

Ques Determine eleven points on a bezier curve with equidistant parametric values control points  $(x_0, y_0) = (50, 180)$ ,  $(x_1, y_1) = (250, 100)$ ,  $(x_2, y_2) = (600, 300)$  &  $(x_3, y_3) = (500, 50)$  distributed over a screen of resolution  $640 \times 350$ .

Sol, standard bezier curve is given by;

$$P(u) = [x(u), y(u)]$$

$$= \sum_{k=0}^n \frac{n!}{(n-k)!k!} \cdot u^k (1-u)^{n-k} \cdot p^k$$

control points - 4.

$$\text{Degree} = 4-1 = 3 = n.$$

$$x(u) = \sum_{k=0}^3 \frac{3!}{(3-k)!k!} \cdot u^k (1-u)^{3-k} \cdot x_k$$

$$= (1) u^0 (1-u)^3 x_0 + 3u(1-u)^2 \cdot x_1 + 3u^2(1-u) x_2 + 1 \cdot u^3 x_3$$

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$$x(u) = 50(1-u)^3 + 750(1-u)^2 + 1800u^2(1-u) + 500u^3$$

$$y(u) = \sum_{k=0}^3 \frac{3!}{(3-k)!k!} u^k (1-u)^{3-k} \cdot y_k$$

$$= 1u^0(1-u)^3 y_0 + 3u(1-u)^2 \cdot y_1 + 3u^2(1-u) y_2 + 1 \cdot u^3 \cdot y_3$$

$$= 180(1-u)^3 + 300u(1-u)^2 + 900u^2(1-u) + 50u^3$$

For every point,  $u$  must be varied from 0 to 10 stepping.

Point	$u$	$x(u), y(u)$
1	0	(50, 180)
2	0.1	(113.9, 163.67)
3	0.2	(183.2, 159.76)
4	0.3	(254.3, 163.89)
5	0.4	(323.6, 171.68)
6	0.5	(387.5, 178.75)
7	0.6	(442.4, 180.72)
8	0.7	(484.7, 173.2)
9	0.8	
10	0.9	

construct a Bézier curves for the points.

$$P_0(4,2) \quad P_1(8,8) \quad P_2(16,4)$$

Sol

$$\sum_{k=0}^2 P_k B_{k,n}(u) \quad 0 \leq u \leq 1$$

$$P_k \frac{n!}{(n-k)!k!} u^k (1-u)^{n-k}$$

$$P_0 B_{0,2}(u) + P_1 B_{1,2}(u) + P_2 B_{2,2}(u)$$

$$\rightarrow x(u) = x_0 B_{0,2}(u) + x_1 B_{1,2}(u) + x_2 B_{2,2}(u)$$

$$\rightarrow y(u) = y_0 B_{0,2}(u) + y_1 B_{1,2}(u) + y_2 B_{2,2}(u)$$

$$B_{0,2}(u) = \frac{n!}{k!(n-k)!} u^k (1-u)^{n-k} = \frac{2!}{2!0!} = 1 \cdot u^0 (1-u)^2$$

$$= (1-u)^2$$

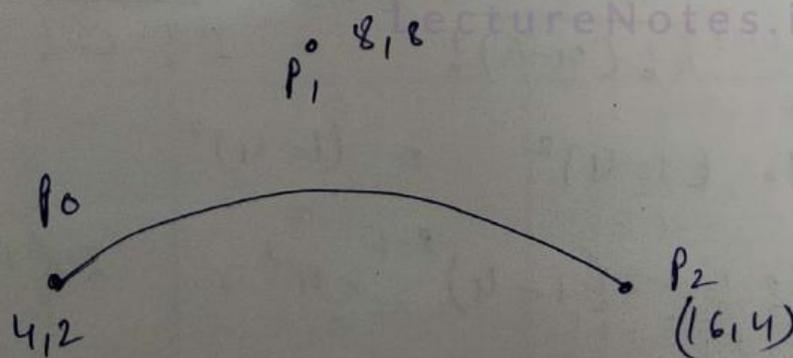
$$B_{1,2}(u) = \frac{2!}{2!1!} (1-u)^{2-1} \cdot u^1 = 2u(1-u)$$

$$B_{2,2}(u) = \frac{2!}{2!0!} u^2 (1-u)^0 = u^2$$

$$\begin{aligned}
 x(u) &= 4(1-u)^2 + 8(1-u)2u + 16u^2 \\
 &= 4 + 4u^2 - 8u + 16u - 16u^2 + 16u^2 \\
 &= 4 + 4u^2 + 8u.
 \end{aligned}$$

$$\begin{aligned}
 y(u) &= 2(1-u)^2 + 8(1-u)2u + 4u^2 \\
 &= 2 + 2u^2 - 4u + 16u - 12u^2 \\
 &= -10u^2 + 12u + 2.
 \end{aligned}$$

$u$	$x(u)$	$y(u)$
0	4	2
0.2	5.76	4
0.4	7.84	5.20
0.6	10.24	5.6
0.8	12.96	5.2
1.	16	4.



# Example for Bezier Curve

Ques. A cubic Bezier curve segment is described by control points  $P_0(2,2)$ ,  $P_1(4,8)$ ,  $P_2(8,8)$ ,  $P_3(9,5)$ . Another curve segment is described by  $Q_0(a,b)$ ,  $Q_1(c,2)$ ,  $Q_2(15,2)$  &  $Q_3(18,2)$ . Determine the value of  $a, b, c$  &  $\infty$  that curve segment join smoothly.

Sol. For  $C^0$  continuity at the point of contact  $P_3(9,5)$  &  $Q_0(a,b)$  must be same.

$$P_3(9,5) = Q_0(a,b)$$
$$a=9, b=5$$

As the curve join smoothly at the point of coincident  $C^1$  continuity at this point should be same.

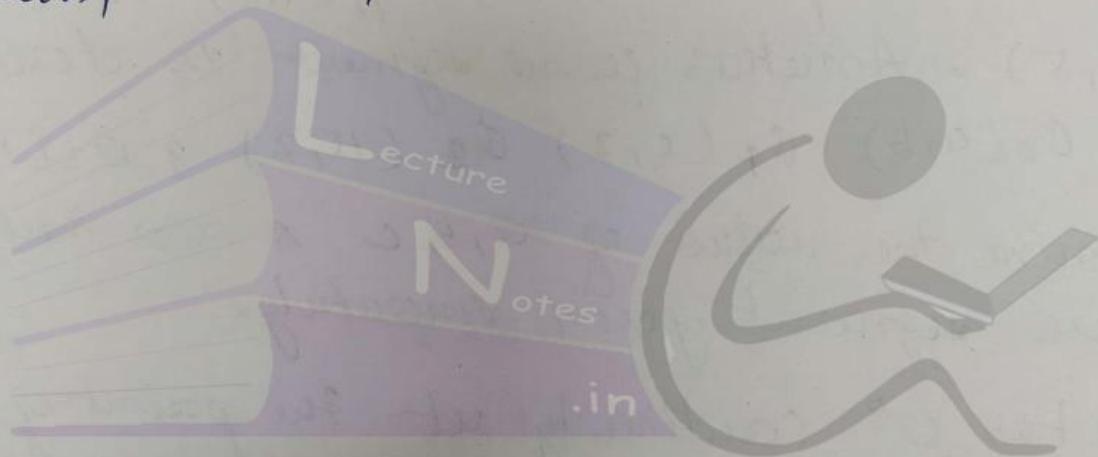
Thus,  $P_3' = Q_0'$  that the tangent vectors at the end point of the first curve should be identical to the tangent vectors at the starting point of the second curve.

$$P_3 - P_2 = d_1 - d_0$$
$$\{ (9, 5) - (8, 8) \} = \{ (c, 2) = (a, b) \}$$

$$(c, 2) = 2(9, 5) - (8, 8)$$
$$= 10, 2$$

$$c = 2.$$

Thus,  $a = 9$ ,  $b = 5$ ,  $c = 2$ .



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# *Computer Graphics And Multimedia*

Topic:

*B-Spline Curve And Its Properties*

Contributed By:

*Ankita Jiyani*

# B-Spline Curve & it's Properties

## Properties of B-spline curves; Basis-splines.

1. The sum of the B-spline basis functions for any parameter value is 1.
2. The max. order of the curve is equal to the no. of vertices of defining polygon.
3. The degree of B-spline polynomial is independent on the no. of vertices defining polygon.
4. B-spline allows the local control over the curve surface because each vertex affects the shape of a curve only over a range of parameter values where its associated basis function is non zero.
5. The curve generally follows the shape of defining polygon.

6. B-spline curve is made of  $n+1$  control points & order of the curve is  $k$ , where  $k$  is the segments form in the curve. The value of  $k$  is decided by the user.

Suppose, if we have segments no,  $k$  then it will give us the polynomial of degree  $k-1$ .

$$k=3, \text{ degree} = 2$$

$$k=4, p(x) = x^3$$

$$2 \leq k \leq n+1$$

7. The no. of segments,  $n-k+2$ .

$$n=6, \text{ control point} \rightarrow 7 (n+1)$$

$$\text{segment}; k=3.$$

$$6 - 3 + 2 = 5$$

Total segments = 5.

variable,  $u \rightarrow 0$  to 5.

8. B-spline curve allow us to change the no. of control points without changing the degree of the polynomial.

# Blending Function for B-Spline Curve

Blending Function for B-spline curve :-

$$P(u) = \sum_{i=0}^n B_{i,k}(u) P_i$$

where,  $P_i(x_i, y_i, z_i)$

$$x(u) = \sum_{i=0}^n B_{i,k}(u) x_i$$

$$y(u) = \sum_{i=0}^n B_{i,k}(u) y_i$$

$$\begin{cases} 0 \leq u \leq n-k+2 \\ 0 \leq k \leq n+1 \end{cases}$$

$$B_{i,k}(u) = \frac{u - t_i}{t_{i+k-1} - t_i} B_{i,k-1}(u) + \frac{t_{i+k} - u}{t_{i+k} - t_{i+1}} B_{i+1,k-1}(u)$$

$t_i$  ( $0 \leq i \leq n+k$ ) - knot value.

$$t_i = 0 \quad \text{if } i < k$$

$$t_i = i - k + 1 \quad \text{if } k \leq i \leq n$$

$$t_i = n - k + 2 \quad \text{if } i > n$$

$$B_{i,k}(u) = 1 \quad \text{if } t_i < u \leq t_{i+1}$$

$$= 0 \quad \text{otherwise}$$

Ques Construct a B-spline curve, with 6 control points & 3 segments.

$$n+1 = 6 \\ n = 5, \quad k = 3$$

$$0 \leq i \leq 5+3 = \underline{0 \leq i \leq 8}$$

control values.

$$P(u) = B_{i,k}(u) p_i$$

For  $B_{0,3}$ ,  $t_0 = 0$ ,  $0 < 3$ ,  $i < k$ .

For  $B_{1,3}$ ,  $t_1 = 0$ ,  $1 < 3$ ,  $i < k$

For  $B_{2,3}$ ,  $t_2 = 0$ ,  $2 < 3$ ,  $i < k$

For  $B_{3,3}$ ,  $t_3 = 1$ ,  $i - k + 1$ ,  $3 - 3 + 1$ ,  $k \leq i \leq n$ .

For  $B_{4,3}$ ,  $t_4 = 2$ ,  $i - k + 1$ ,  $4 - 3 + 1$ ,  $k \leq i \leq n$ .

For  $B_{5,3}$ ,  $t_5 = 3$ ,  $i - k + 1$ ,  $5 - 3 + 1$ ,  $k \leq i \leq n$ .

For  $B_{6,3}$ ,  $t_6 = 4$ ,  $n - k + 2$ ,  $5 - 3 + 2$ ,  $i > n$

For  $B_{7,3}$ ,  $t_7 = 4$ ,  $n - k + 2$ ,  $5 - 3 + 2$ ,  $i > n$

For  $B_{8,3}$ ,  $t_8 = 4$ ,  $n - k + 2$ ,  $5 - 3 + 2$ ,  $i > n$

$$B_{0,3}(u) = (1-u)^2 \underline{N_{2,1}(u)}$$

# Difference between Bezier and B-Spline Curves

Difference b/w Bezier & B-spline curve.

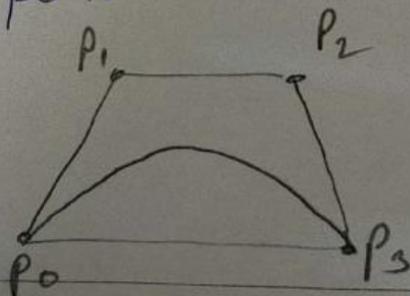
**Bezier curve**

It has global control over the curve. Changing one control point affects whole curve.

The degree of curve depends on the no. of control points.

$$\text{degree} = n - 1$$

It is defined as curve lies in the convex hull of their control points.



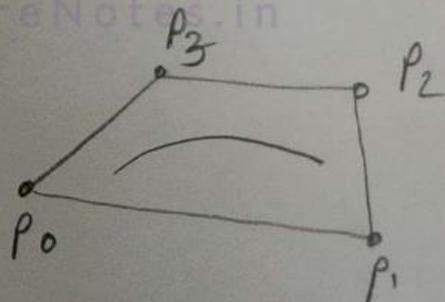
**B-spline curve**

It has local control over the curve. Changing one control point affects only one part of the curve.

The degree of curve depends on the no. of segments.

$$\text{degree} = k - 1$$

It is defined as curve lies within the convex hull of their control points.



## Bezier curves

The curve touches it first & last control points.

The range of  $u$ , lies  
b/w,  $0 \leq u \leq 1$

## B-spline curves

The curve here is not necessarily touches the first & last control points.

The range of  $u$ , lies.  
b/w  $a$   $0 \leq u \leq n-k+2$   
 $0 \leq k \leq n+1$

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# *Computer Graphics And Multimedia*

Topic:  
*Illumination Model*

Contributed By:  
*Ankita Jiyani*

# Illumination Model

Illumination Model :-

- ↳ It is used to calculate the intensity of light of a point on the surface of an object.
- ↳ It is also known as lighting model or shading model.
- ↳ The calculation done by the model is used by surface rendering method to determine the light intensity for all projected pixels for various surfaces in a scene.
- ↳ The model will calculate the intensity projected from a particular surface point in a specified viewing direction using the following parameters.
  - i) optical properties of a surface.
  - ii) The relative positions of the surfaces in a given scene.

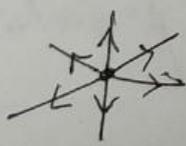
- iii) The color & positioning of light sources.
- iv) positions & orientation of the viewing plane.

Light sources :-

- ↳ light source is the source which emits the light.
- ↳ when we view an opaque object, we see the reflected light from the surface of the object.
- ↳ total reflected light = light directly from light source / light emitting source.
- + light from reflecting surface / light reflecting src

Point source :-

↳ Rays from the source follow radially diverging paths from the source.



↳ point source dimensions are small as compared to size of the objects in the scene.

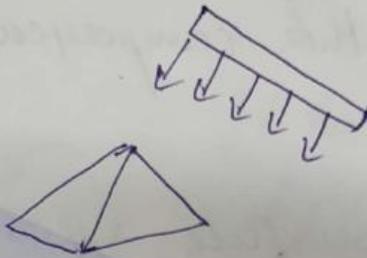
Zg :- Sun (sufficiently far from the scene can be accurately modeled as point source).

# Types of Light Sources

Distributed light source:

A screen of the source is not small compared to the surfaces in the scene.

eg: A fluorescent light



Parallel light source:

A source of light which is far away and the rays coming from the source is considered as parallel light source.

There are 3 types reflection:-

1. Ambient reflection:-

The light is coming from different source. And the light is reflected rays from other object.

Or, The object is visible when nearby objects are illuminated.

$$\text{Intensity} = I_a K_a$$

$0 \leq K_a \leq 1$   $I_a \rightarrow$  Intensity of Reflection  
 $K_a \rightarrow$  Diffuse Reflectivity coefficient.

Diffuse Reflection :-

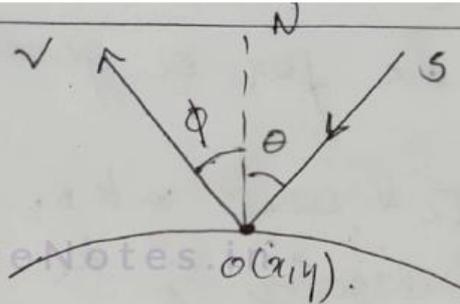
Reflection from the rough surfaces are scattered. They are scattered in all directions.

Eg: A blue object illuminated by a white light source reflects the blue component of the white light & absorbs all other component.

Specular Reflection :-

The shiny surface appearance, when light is reflected from any object is called as specular reflection.

or Highlights or bright spots appearance etc.  
Specular reflection.  
or Mostly found on shiny surfaces



$$\text{Intensity of spec} = f(\vartheta_r, \theta, \rho, \phi) + A(v)$$

$\vartheta_r \rightarrow$  intensity of light source.

$\rho \rightarrow$  reflectivity coefficient

$A(v) \rightarrow$  Ambient light vector.

For ideal diffuse reflection;  
 the light coming from light source,  
 after hitting any surface, part of it reflects  
 back at same angle. see discarding  
 $\phi$  from above function.

$$I_{\text{diff}} = f(\vartheta_r, \theta, \rho) + A(v)$$

Also known as, Lambertian reflector.



# *Computer Graphics And Multimedia*

Topic:

*Lambert's Cosine Law*

Contributed By:

*Ankita Jiyan*

## Lambert's Cosine Law

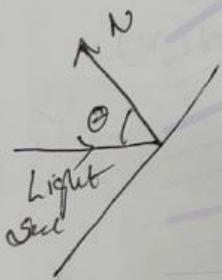
Lambert's cosine law :-

How much light the surface receives from a light source depends on the angle b/w its normal and vector from the surface point to the light.

Lambert's law : The radiant energy  $I_d$  from a small surface  $dA$  for a given light source is :-

$$I_d = I_L \times \cos \theta \times k_d$$

$I_L$  - Intensity of the light source.  
 $\theta$  - angle b/w the surface normal  $N$  & light vector  $L$ .



↳ A surface is illuminated by a point source only if the angle of incidence is in the range  $0^\circ$  to  $90^\circ$ , i.e.  $0 \leq \cos \theta \leq 1$

↳ when  $\cos \theta$  is -ve, the light source is behind the surface.

↳ The total reflection is :- ambient + diffuse

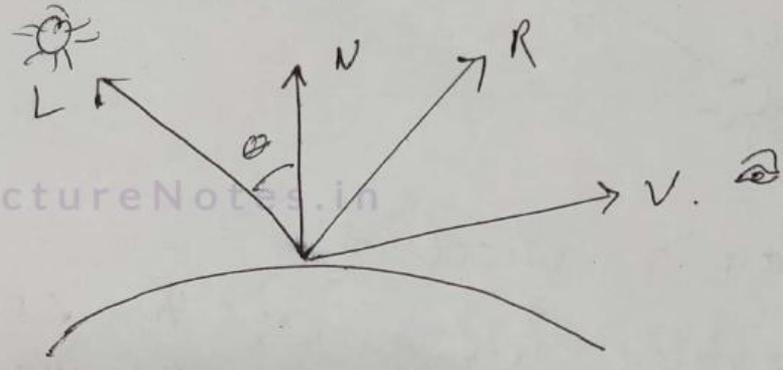
$$I_{diff} = k_a \times I_a + k_d \times I_L \times (N \cdot L)$$

$$\cos \theta = N \cdot L$$

$N$  → Normal vector to surface

$L$  = the unit direction vector to the point light source

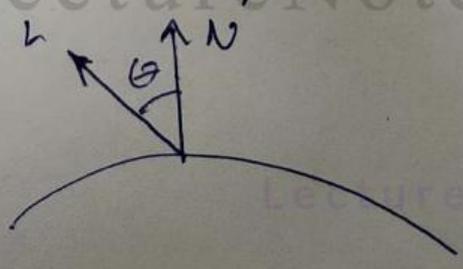
$$0 \leq kd, \quad ka \leq 1$$



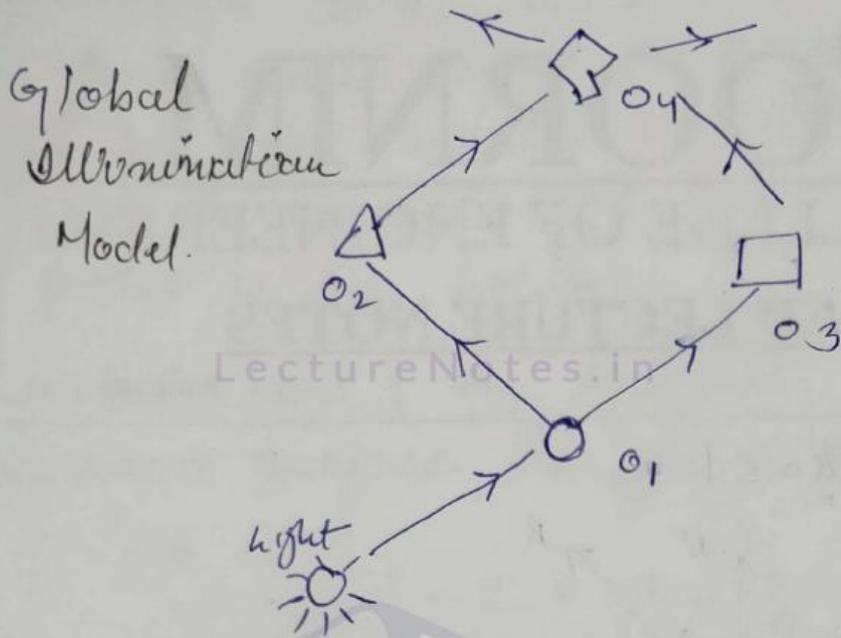
### Types of Illumination Model :-

Global Illumination model. - It takes into account the interaction of light from all the surfaces in the scene.

Local illumination model, only considers the light, the observer position & object material properties.



# Global Illumination Model



Illumination vs shading.

→ Illumination determines the color of a surface point by simulating some light attributes.

↳ Shading model applies the illumination model at a set of points & color the whole image.



# *Computer Graphics And Multimedia*

Topic:

*Shading And It's Types*

Contributed By:

*Ankita Jiyani*

# Shading and its types

Shading : By using intensity calculated by illumination model, we need to color the whole object.

Types of Shading :-

① Flat shading or constant shading.

It is a technique used in 3D graphics. It shades each polygon of an object based on the angle between the polygon's surface normal & the direction of light source, their respective colors & the intensity of light source.

It is usually used for high speed rendering where more advanced shading techniques are too computationally expensive.

Disadvantage :-

- It produce unrealistic look.
- It produce mesh band effect.

Page No.:

Flat shading of polygon facets provides an accurate rendering for an object if set of the following assumptions are valid :-

- 1) The object is a polyhedron & is not an approximation of an object with a curved surface.
- 2) All light sources illuminating the object are sufficiently far from the surface so that  $D.O.L$  & the attenuation function are constant over the surface.
- 3) The viewing position is sufficiently far from the surface so that  $V.R$  is constant over the surface.

### Application :

- The algorithm is applied to the scene when both light source & viewer are far distant from the object.
- To display fast moving object in a scene, this algorithm is applicable.

# Gouraud Shading

The mach band effect occurs, if intensity varies uniformly.

Gouraud shading overcomes this limitation.

## Gouraud Shading :-

- ↳ It is an interpolation technique developed by Henri Gouraud in 1970.
- ↳ Intensity levels are calculated at each vertex and interpolated across the surface of a polygon.
- ↳ It eliminates the discontinuities of the intensity, as intensity values for each polygon are matched with the values of adjacent polygons along the common edges.
- ↳ To render a polygon, following steps are performed;

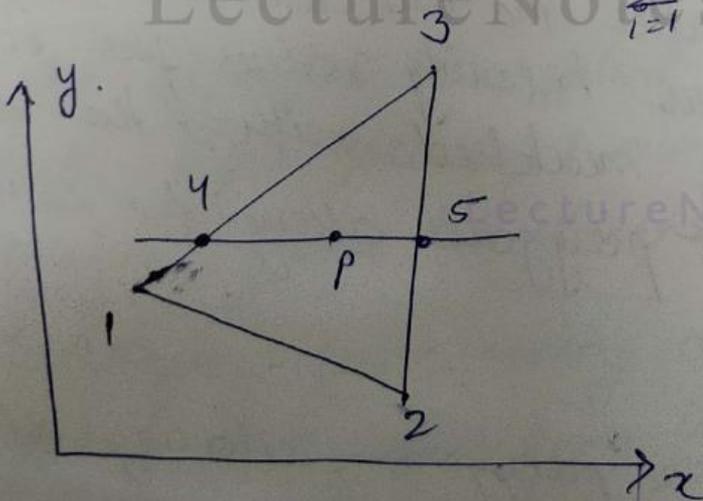
- 1) Determine the average unit normal vectors at each polygon vertex.
- 2) Apply an illumination model to each vertex to calculate the intensity of the vertex.
- 3) Interpolate the vertex intensity in linear manner, over the surface of the polygon.

The average unit normal vector at  $v$  is given

as :-

$$N_v = \frac{N_1 + N_2 + N_3 + N_4}{|N_1 + N_2 + N_3 + N_4|}$$

More generally :-  $N_v = \frac{\sum_{i=1}^n N_i}{\sum_{i=1}^n |N_i|}$



The intensity at point 4 can be interpolated from intensity 1 & 3.

$$I_4 = \frac{y_4 - y_3}{y_1 - y_3} I_1 + \frac{y_1 - y_4}{y_1 - y_3} I_3$$

The intensity at point 5 can be interpolated from intensity 2 & 3.

$$I_5 = \frac{y_5 - y_2}{y_3 - y_2} I_3 + \frac{y_3 - y_5}{y_3 - y_2} I_2$$

The intensity at point p, can be interpolated from intensity 4 & 5.

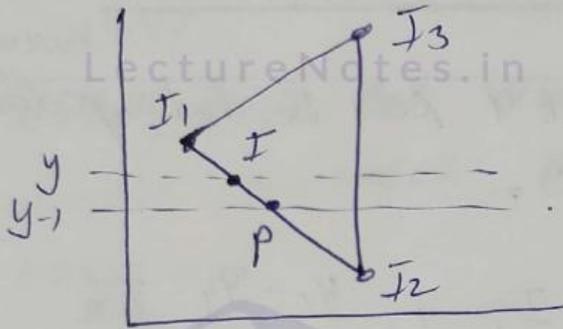
$$I_p = \frac{x_5 - x_p}{x_5 - x_4} I_4 + \frac{x_p - x_4}{x_5 - x_4} I_5$$

If the intensity at edge position  $(x, y)$  is interpolated as:

$$I = \frac{y - y_2}{y_1 - y_2} I_1 + \frac{y_1 - y}{y_1 - y_2} I_2$$

then we can obtain the intensity along this edge for the next scan line  $y-1$  as:

$$I' = I + \frac{I_2 - I_1}{y_1 - y_2}$$



Advantages:-

It removes the discontinuities associated with flat shading model.

Disadvantages:-

Highlights on the surface are sometimes displayed with anomalous shape, and linear interpolation may cause bright or dark intensity streaks (Moiré bands) to appear on the surface. This is overcome by Phong shading.

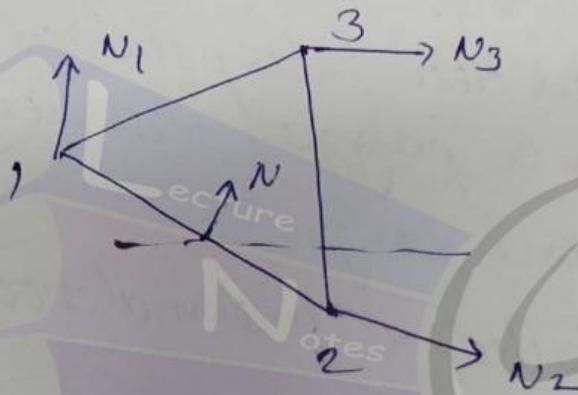
# Phong Shading

## PHONG SHADING.

- A more accurate interpolation based approach for rendering a polygon was developed by Phong Bui Tong.
- ↳ It is also called as normal vector interpolation rendering.
- ↳ It interpolates normal vectors instead of intensity values.
- ↳ The rendering of a polygon is performed by using following steps:-
  - 1) Determine the average unit normal vectors at each vertex of the polygon.
  - 2) linearly interpolate the vertex normal over the projected area of the polygon.

3) Apply an illumination model at positions along scan lines to calculate pixel intensities using the interpolated normal vectors.

- The normal vector  $N$  for the scan line intersection point along the edge b/w vertices 1 & 2 is obtained as:-



$$N = \frac{y - y_2}{y_1 - y_2} N_1 + \frac{y_1 - y}{y_1 - y_2} N_2$$

- Incremental methods are used to evaluate normals between scan lines & along each individual scan-line.

↳ At each pixel position along a scan line, the illumination model is applied.

### Advantages :-

- Intensity calculations are more accurate
- Displays more realistic highlights on a surface.
- Reduces Moiré Band effect.
- Suitable for shiny surfaces.

### Disadvantages :-

It requires more calculations and greatly increases the cost of shading steeply

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